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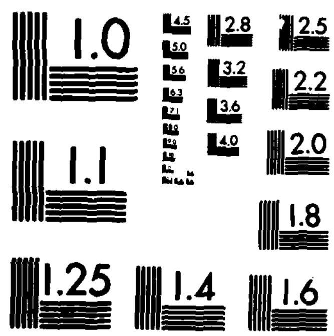

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A STUDY OF ANALYTIC MODELING TECHNIQUES FOR LANDING GEAR DYNAMICS

Dr. Stephen M. Batill
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May 1982

Final Report for Period November 1980 - December 1981

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The ability to accurately predict the dynamic response of an aircraft while it is operating in the taxi mode depends, in part on the correct modeling of the dynamic characteristics of the landing gear system. Traditionally, landing gear have been designed to absorb landing impact ("shock absorber") and their characteristics during periodic, oscillatory response ("spring") have been considered as secondary. With the increased emphasis on the rough or damaged field taxi operation, there is a requirement to determine the

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best methods for modeling the gear system. This report documents a brief review of the state of the art of gear modeling. A study was then conducted to evaluate important model parameters, using a simple cantilevered gear computer simulation. Also included is the development of a technique for the experimental determination of important gear system parameters.

PREFACE

The work documented in this report was performed by the Department of Aerospace and Mechanical Engineering, University of Notre Dame, for the Department of the Air Force, Air Force Wright Aeronautical Laboratories (AFSC), Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract F33615-80-K-3242. The work was monitored under Program Element 61102F, Project 2307, Task 2307N5 and Work Unit 2307N520. The Principal Investigator on the project was Dr. Stephen M. Batill.

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The report documents work conducted from November 1980 to December 1981. The final report was submitted in February 1982.

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SUMMARY

The ability to accurately predict the dynamic structural response of an aircraft while it is operating in the taxi mode depends, in part, on the correct modeling of the dynamic force characteristics in a landing gear system. Traditionally, analyses have concentrated on the prediction of landing impact loads since these loads were considered to be the greatest loads which would be experienced by the landing gear system, and the consideration of landing gear loads during the periodic, oscillatory response associated with the taxi mode of operation has been considered as secondary. However, with the increased emphasis on rough or damaged field taxi operation, there is a requirement to more accurately describe the component landing gear forces for the purpose of improving aircraft dynamic structural response predictions. This report documents a brief review of the state of the art of modeling landing gear forces. A study was then conducted to evaluate important model parameters, using a simple cantilevered gear computer simulation. Also included is the development of a technique for the experimental determination of improved important gear system parameters.

SECTION I

INTRODUCTION

The term "landing gear" when applied to modern commercial and military aircraft refers to that system which includes tires, wheels, brakes, struts and related control and retraction equipment. This complex system evolved from simple coil or leaf springs to the oleo-pneumatic systems used today. The gear obviously plays an important role in the operation of an aircraft system, both on the ground and, indirectly, while in the air. Its influence on ground "performance" is the primary focus of this report but, because the gear contributes to a significant portion of the structural weight of an aircraft, it also influences flight performance. Gear systems in modern aircraft have been designed using materials and techniques that have resulted in lighter, more flexible gear systems. The current generation of aircraft has gear systems varying from 3% to 8% of the total vehicle weight, depending on aircraft type and mission. Requirements for future systems are aimed towards reducing these percentages. Together with this emphasis on reduced weight, there is increased interest in improving the operation on damaged or rough fields. Obviously, the requirements for reducing weight and improving "performance" are not complementary. Therefore, it is necessary to have the ability to accurately predict the gear system performance during all phases of its operation.

The purpose of the landing gear system can be summarized as:

1. Provide support and control for the aircraft while it operates in the taxiing mode, both before take-off and after landing.
2. Absorb the energy associated with landing impact.

The gear must provide for the safety of the aircraft, crew and cargo for each of these conditions. The gear system interacts with the ground and transmits forces from the ground to the airframe. It is the goal of the gear design to minimize any adverse effects due to this interaction.

1. PROBLEM OVERVIEW

When operating in the taxi mode, the aircraft structural loads and dynamic response are governed by the forces and deflections applied at the gear attachment points. The internal loads and response of the gear system are the results of the contact with both the ground and with the aircraft. Conditions critical to the gear structural design may not be particularly severe with respect to the loads transmitted to the aircraft. Similarly, certain conditions which may not be particularly severe with regard to the gear structure may bring about critical conditions within the airframe. Much of the emphasis in gear system design has focused on the second of the two purposes listed above - landing impact. As a result, most gear systems are designed to be very effective as energy or shock absorbers for a single impact loading. The loads transmitted to the airframe for such an impact may be critical in certain areas of the structure, both due to their magnitude and repetition (i.e., fatigue). There are numerous reports documenting the response of both the gear system and the aircraft to landing impact. The emphasis in this area is due to the fact that it was this condition which often resulted in the most critical gear loading on the airframe. Design criteria have been expressed in terms of maximum sink rates at impact, and analyses performed using methods ranging from the simple spring mass system to complex dynamic models of both gear and aircraft systems.

Whereas the overall geometry of the gear system (i.e., whether cantilevered, levered, allowable stroke, size, etc.) may be dictated by the landing impact condition, certain phases of the first of the two purposes mentioned earlier may often present the most critical design conditions for the gear structure. Ground operations such as braking, towing, and shimmy often determine the most critical loads on the gear itself. Description and prediction of the loads for these conditions can be quite complex and depend to a great extent on the type of gear, wheel and tire mechanism, as well as the type of surface on which the aircraft is operating.

For many current aircraft, there is another ground operation condition which can influence both the gear and the aircraft structural design - the dynamic taxi condition. Current specifications require the aircraft to be able to operate on a runway with a specified roughness distribution and to have the ability to successfully encounter certain types of simple bumps and dips. To predict the dynamic response of the gear system and the aircraft, the accurate representation of the gear's dynamic characteristics is necessary. A number of techniques, to be briefly described later in this report, have been developed to describe the dynamic taxi response. The methods used to model the dynamic response of the gear are, for the most part, the same as those used to predict response during landing impact.

This gives rise to the problem which is the topic of this report. What are the limitations of the models and prediction methods used to evaluate gear and aircraft dynamic response during dynamic taxi operation? This becomes a particularly important question when one considers the near failure operation of gear and aircraft systems in the case of "one" time operation on a severely rough or damaged runway surface. Are those methods which have proved to be adequate for describing the impact response sufficient to describe the large amplitude, oscillatory (though aperiodic) response of the highly nonlinear gear/aircraft system?

2. PURPOSE OF STUDY

The purpose of this report is to provide a preliminary assessment of the current state-of-the-art of the modeling of the gear system as it is used in dynamic taxi analysis and to present a discussion of a technique for improving the prediction of critical gear parameters. The report is presented in three sections, the first being a presentation of results of an industry survey in which both gear and aircraft manufacturers were asked to comment on current methodology and problems. The second is a presentation of the result of a computer simulation study of the influence of a number of important characteristic parameters on the dynamic response of a simple, single-chambered, cantilevered gear. The third section is the development of a parameter identification procedure which may prove useful in future experimental assessment of the dynamic characteristics of gear systems.

SECTION II

REVIEW OF CURRENT MODELING TECHNIQUES

The following review of current modeling methods used for the prediction of landing gear system influence on aircraft dynamics while operating in the taxi mode is an overview at most. The overall dynamic system (i.e., aircraft, gear, tire, runway, etc.) is extremely complex and a complete description of details of even the gear/tire system is beyond the scope of this report. In most conventional applications the gear is a device whose dynamic response is determined by the operation of an energy-absorbing mechanism, usually an oleo-pneumatic strut and a spring-like, deformable tire. The gear geometry can vary from a simple cantilever with a single tire to a complex, articulated mechanism with multiple tire assemblies. The purpose of this report is to look with some detail at the dynamics of a single-chambered, oleo-pneumatic strut and simple "spring" tire. Although this may appear to be an extreme simplification, the operation of such a mechanism is characteristic of many gear assemblies.

1. LITERATURE REVIEW

The type of analytic model used to represent the influence of the landing gear system on the dynamics of an aircraft operating in the taxi mode depends upon the purpose and complexity of the analysis being performed. Most taxi analyses can be classified into two categories. In the first, the aircraft and gear system is modeled as a multiple degree of freedom "lumped" mass system and the response of this dynamic system is determined by either an analog or digital numerical "step-by-step" integration of the equations of motion.[1-9] This integration is performed to determine the response to a given runway profile and, therefore, must be repeated for different profiles or different taxi conditions. This approach allows for the detailed analytic modeling of any or all of the parts of the dynamic system and often incorporates nonlinear effects such as friction, damping, airplane aerodynamics and active control systems. This type of analysis is often used to evaluate the landing impact response and may also be used to help design parts of the gear system for impact loading. It is this method of analysis which will be the focus of the current report. The second method uses a linearized model of the aircraft and gear system, and a statistical or power spectrum description of the "typical" runway surface.[10-16] Such a technique has the disadvantage of requiring a "linearized" model of the system dynamic characteristics. As will be noted later this may be an undue limitation, particularly when considering large amplitude dynamic response. The results of such an analysis are also usually presented as average or "RMS" response. An advantage of this approach is that the sensitivity of the overall response to particular parameters can be determined and generalized optimization studies performed.[17] Each of these two approaches has its own respective advantages and disadvantages but, in each case, the accurate prediction of the aircraft system response requires an accurate model for the landing gear contribution.

There appears to have been little work related to aircraft landing gear dynamics prior to 1940. This is primarily due to the use of relatively low air pressure in tires which were the main "bump" absorbing devices on the aircraft. With World War II came heavier and faster landing aircraft and, thus, an increase demand on landing gear performance. Both the U.S. and Germany spent considerable effort in developing a better understanding of the landing impact problem, although there appeared to be little interest in the taxi dynamics area. Flugge's paper[18] outlines some of the early German developments. The U.S. advances in the World War II and post war era are highlighted in References 18-23. Probably the most significant and often referenced work is that of Milwitzky and Cook[23] which presents the basic analytic model used in many dynamic simulations.

Milwitzky and Cook's model was developed to describe the physics of the oleo-pneumatic strut due to impact loading. It was not mathematically sophisticated but included nonlinearities which required, at that time, lengthy mechanical numerical calculations when applied to the landing impact problem. Therefore, the use of an analytic description of this type was limited for a practical analysis. Fortunately, it was about the same time that both analog and digital computing machines were evolving and they opened the door for rapid solution of the nonlinear differential equations of motion. Much of the emphasis since these developments in the early 1950's has been on the application of this type of gear model in more detailed studies for particular aircraft systems. Such emphasis has involved the inclusion of detailed flexible aircraft models but the gear models have remained relatively simple. The author was unable to locate any recent studies in which improvements of the gear model itself was the primary concern. This was particularly true in the case of any experimental work conducted to confirm the accuracy of the basic gear model. To help in the evaluation of this representation of the gear characteristics, the next few paragraphs will present an overview of this analytic description.

The three basic mechanisms described in Reference 23 are common to the gear model in most taxi simulations. There is the "air" spring used to describe the reaction of compressed gas contained in the strut due to a change in gas volume as the strut compresses or extends; there is a hydraulic force generated as fluid is forced through an orifice or group of orifices as a result of strut motion; and there are frictional forces created as bearing surfaces slide relative to each other during strut motion. It is through the action of these three mechanisms that forces are transmitted from the lower strut which is attached to the wheel, to the upper strut which is attached to the aircraft. These are internal forces that occur as equal and opposite pairs as they act on the upper and lower sections of the strut. Using the typical oleo-pneumatic strut shown schematically in Figure 1, each of these force mechanisms will be discussed briefly.

a) Air Spring

As the strut expands or contracts, the volume of the trapped gas must change and, thus, the pressure exerted by the gas on the strut is a function of the stroke position of the strut. The gas pressure is usually related to the chamber volume by the polytropic compression law:

$$pv^n = \text{constant}$$

where p = upper chamber gas pressure,

v = upper chamber gas volume

and n = polytropic gas constant.

The choice of the value for n ranges from 1.0 for an isothermal compression to about 1.4 for an adiabatic process. An actual dynamic compression is neither adiabatic nor isothermal, and the selection of a value for n appears to be more of an art than a science. Reference 23 discusses some of the limitations on the use of a constant value for n for even a single impact, and to assume a constant value during a complete taxi run is probably more difficult. Some simulations use a different, though constant, value of n for extension and compression.[6]

Although there has been some work done to determine the details of the gas compression process,[21-25] all correlations seem to be with limited drop test data. There appears to be a need for the experimental verification of the details of the gas compression process and a determination of those parameters which may influence this process.

There are a number of other phenomena that would be considered as second order effects but are related to the behavior of the air spring; these are hydraulic fluid compressibility, fluid-gas mixing and structural deformation (i.e., expansion) of the gear chamber. In many cases these effects are included by either modifying the "v" (volume) or "n" in the equation cited. Although, computationally, it is rather early to include such second order effects by modifying either v or n, there appears to be no definite procedure for determining how these parameters vary.

b) Hydraulic Damping

As the gear strokes, hydraulic fluid is forced through an orifice or group of orifices. The hydraulic losses associated with the flow through these constrictions result in pressure differences between the regions above and below the orifice plates. The pressure difference creates a hydraulic force which opposes the relative motion of the upper and lower strut. The magnitude of the force can be written as

$$F_{hy} = \frac{K}{C_d^2}\,\dot{s}^2$$

where F_{hy} = hydraulic force,

K = constant related to hydraulic fluid density, chamber, and orifice geometry,

C_d = orifice discharge coefficient

and $\dot{s}$ = stroke rate of gear.

This model is based on a steady, one-dimensional turbulent (i.e., high Reynolds number) flow of the hydraulic fluid through the orifice. The discharge coefficient, C_d, is a function of flow characteristics through the orifice and its geometry (i.e., sharp edged, rounded, etc.). Depending on the stroke rate, the hydraulic resistance can be a major part of the total gear force. This is the case for landing impact. A metering pin may be used to alter the size of the orifice to "tailor" the hydraulic force for a specific operating condition. Usually, the metering pin is designed based on landing impact load requirements for the aircraft. Multiple orifices or extension snubbers which are used to increase the hydraulic forces during the extension stroke and, thus, limit excessive rebound are usually modeled in much the same way. Often, this is accomplished using a different area over which the hydraulic pressure difference acts and a separate snubber orifice coefficient, or simply by modifying the primary orifice discharge coefficient during the extension process.

It appears that experimental verification of the hydraulic force contribution to the total gear force is also lacking. Most of the dynamic taxi simulations seem to use the "$\dot{s}^2$" type of hydraulic damping with constant orifice coefficient. Although other types have been considered,[26,27] they are not so common in dynamic simulation models. Even in those methods where the discharge coefficient is a function of orifice Reynolds number and stroke direction, these parameters are not varied in time during a particular taxi simulation. Reference 23 suggests that even the basic nature of the physics of the hydraulic damping may vary considerably with fluid foaming, as well as other property changes, as "energy" is absorbed into the fluid. The orifice coefficient is one of the parameters used to correlate drop test data with simulation results and its value is often determined from such a correlation. Since, in certain cases, this parameter is found to have a significant influence on the calculated response of a landing gear system, the accuracy of the methods used for its prediction are obviously important.

During the actual operation of the gear, the orifice flow is highly unsteady which may present a serious problem when the "steady" flow hydraulic force model is used for taxi simulations. There appears to be very little literature related to unsteady or oscillatory orifice flows. There has been some work related to unsteady orifice flows conducted with application to heart valve research. One of these studies[28] which coincidentally uses an orifice geometry similar to that of a simple landing gear shows that significant phase lags between pressure differences across the orifice and flow reversals occur. Although the results of such a study are not directly applicable to the landing gear case, they do indicate areas that should be of concern.

c) Frictional Forces

As the upper and lower strut sections move relative to each other, frictional force interactions are generated on bearing surfaces. In numerous simulations such as the one conducted in this report, these forces are ignored simply because the best technique for their inclusion is not obvious. Although the bearings are lubricated by the gear hydraulic fluid, most models follow the suggestions of Reference 23 and the friction force contribution is included as dry or Coulomb friction in the form:

$$F_f = \frac{\dot{s}}{|\dot{s}|} (\mu_1 |F_1| + \mu_2 |F_2|)$$

where μ_1 - coefficient of sliding friction for the upper bearing,
F_1 - normal force on the upper bearing
and μ_2, F_2 - correspond to the lower bearing.

The normal forces are due to the geometry of the gear system and are arrived at from equilibrium considerations relative to the other non-axial loads acting on the gear. Such loads are highly gear-geometry and wheel-load dependent. If the structural deformation of the gear itself is included, as it is in some of the more current and complex models, the determination of the bearing's normal forces becomes more involved. Binding of the gear can be included by establishing both a static and a kinetic friction coefficient and by satisfying the force conditions for relative motion. Frictional effects associated with gear deformation and its influence on bearing forces can have significant effects on simulation results. The inclusion of gear deformation which requires detailed gear structural definition represents an additional level of detail that helps to further complicate the gear modeling problem.

The previous discussion was intended as an overview of some of the characteristics of gear modeling which are included in many of the dynamic impact or taxi simulations. The discussion is obviously not complete and the cited references should be used to provide additional detail. As in all literature reviews, the exclusion of many other documents related to this area was accidental and those mentioned are characteristic of overall developments in this field. Those documents cited in this report refer to many other papers and reports related to this particular problem.

2. INDUSTRY SURVEY

In an attempt to make an assessment of gear modeling techniques in use in industry today, a brief survey of a number of interested aerospace firms was conducted. The survey was short and not intended to be complete but it was hoped to establish any significant differences between current industry practices and methods documented in the open literature. The questions in the survey and the names of those organizations which responded are included in Appendix A. The answers to most of the questions will be presented here in a composite form, without identifying the responding organization since individual responses may not always be characteristic of overall company methods or policies. All respondents did not answer every question so the summary of their answers should not be considered as a majority or concensus.

(1) Have you developed computer simulations to predict the dynamic response of an aircraft while operating in a taxi mode?

All respondents who were involved in the design or analysis of land based aircraft had either developed or had experience in using taxi simulation computer programs. Often, the programs were developed for the design of a specific aircraft system and were used for both dynamic impact and taxi calculations. A number of the documents cited are included in the References. Some organizations indicated that reports on the current versions of their simulations were being prepared and should be available within the year. This appears to be an indication of renewed interest in certain aspects of this problem.

(2) What kind of analytic model was used to predict the tire/wheel/gear contribution to the dynamic response?

The tire, although it may represent one of the most critical and complex force transmitting mechanisms in this entire problem, is usually represented as a simple spring with point contact to the ground surface. Both linear and nonlinear springs, some with linear viscous damping, are used. The stiffness characteristics are those provided by experiment or manufacturers' static load deflection curves. Tire bottoming was also mentioned but the manner in which this was incorporated into the model was not detailed. More complex tire models were used for some ground-based, off-road vehicle dynamic simulations, and included radial segmented rings and deformable ring models; however, it appears that these have not been used for the aircraft simulations. No tire dynamic effects such as variable footprint or spin up were mentioned.

The model outlined in the previous section was cited in each of the survey responses, with only slight modifications. Air spring influence was included as a polytropic compression process with fixed exponential constant. Velocity squared, metered damping was used, in many cases with snubbing, to model the hydraulic force contribution. Bearing friction forces are included in a number of models but influence on the bearing forces due to strut structural deformation was only mentioned as an item which was not, but should be, included. Strut bottoming and full extension metal-to-metal contact may also be considered. Fore-aft or lateral gear deformations can be included by modeling the gear as a simple beam, but these deformations did not interact with the axial response of the strut. The wheel/tire is most often included as an "unsprung" lumped mass attached to the bottom of the strut. The use of this gear model, with no significant variation between responding organizations, seems to indicate its ease in application and a relative acceptance of its results.

(3) Have you performed an evaluation of the analytic model used for the tire/wheel/gear?

Many respondents indicated that some evaluations had been conducted but they did not comment with any detail on the results of such evaluations. Most of the correlations appear to be with drop test data used to arrive at various gear parameters. It was pointed out that additional verification is necessary but presently not possible due to a lack of adequate experimental data. In some cases, experimental programs are planned to help generate some of these data for particular aircraft systems. This creates additional problems in that, while trying to extract the gear or tire influence from data collected or calculated using whole aircraft tests or simulations, the gear influence is often "meshed" by the complex dynamics of the aircraft.

(4) How significant is the choice of the tire/wheel/gear model in the overall simulation?

Since each of the respondents was basically using the same analytic model, the comments were quite interesting. Based on their individual experiences, their satisfaction or criticism of the model varied. For landing impact conditions, tire models were cited as being less important than the strut model. For taxi conditions, particularly over rough surfaces, the tire model was an important, if not more so, than the strut since, for very short wavelength bumps there is very little strut movement and the tire is required to "absorb" much of the bump. The strut model was generally accepted as adequate if the gear parameters could be identified for a particular gear. Obviously, the better the parameter selection, the better the results of the simulation.

(5) Do you feel the model is adequate for large amplitude dynamic response?

Since the structural response, other than the landing gear, is based on linear methods there are limitations on large amplitude responses such as those which involve tire or strut bottoming. A limitation on the gear model is that some model parameters seem to be functions of amplitude of response; therefore, different parameter sets provide better correlation for different dynamic conditions. The tire model (i.e., point contact) presents obvious problems for very short wavelength, large amplitude roughness.

(6) Does the model have the capability of handling:

(a) Gear deformation?

In most cases it does, although this was limited to fore-aft or lateral beam bending. Any additional detail in the gear structure would require more detail at the aircraft structure interface and this was avoided. There was only one mention of cylindrical expansion of the strut chamber.

(b) Frictional effects?

All simulation models appeared to include a dry bearing friction. Details on the choice of friction coefficients were not provided.

(c) Gas/fluid mixing?

Most of the models allow only for a constant value of the polytropic gas compression coefficient and a constant fluid density. It is recognized that, particularly during taxi, excessive gas/fluid mixing does alter the gear characteristics. There is no method available for predicting or describing the influence of this process on the gear performance. Presently, it can only be considered by evaluating performance over a range of polytropic gas constants or by a modification of gas volume and fluid density. It was often suggested that this is an area requiring additional study and testing.

(d) Drag loads in braking or non-braking operation?

Drag loads and slip are usually included with drag loads related to slip rate. Braking is not usually incorporated into the taxi simulations.

(e) Others you may consider more important.

There were few comments in this area. Some mentioned the inclusion of fluid compressibility in their model and others cited the problems associated with multi-chambered gear designs. Again, the difficulty of interfacing the tire with the ground surface and the geometry of the tire footprint was mentioned. Although no solution to the problem was recommended, it was indicated as an area for concern.

(7) What methods are used for determining the parameters used in the gear model?

Open literature data and company reports, either experimental or theoretical, as well as previous experience, provide a basis for the initial selection of the gear parameters. These parameters are then typically modified based on drop test results so that the simulation matches the drop test time histories.

(8) What are considered the most critical gear parameters?

There was no overwhelming concensus, other than that most of the parameters are important. The kind of application, again, bears strongly on what are considered the critical parameters. For impact, the metering pin geometry and value for the discharge coefficient are very important. For taxi response, the critical parameters depend upon, to a degree, the wavelength, amplitude and frequency of the disturbance. For large amplitude or large wavelength disturbances, the strut parameters must be well-defined. For high frequency or short wavelength disturbances, the tire parameters, typically the tire load-stroke curve, is very important. It was recognized that, for particular cases, small changes in given parameters can yield large variations in the simulation result.

(9) What do you think are the drawbacks or deficiences of the available gear models for predicting tire/wheel/gear contributions?

There was a wide range of responses to this question, divided between problems related to the tire and to the gear. A number of respondents felt that the modeling of the dynamic behavior of the tire was a real "weak link" in the process. Both lateral and rotational tire characteristics were cited as problem areas where little help is available. There is also a serious lack of dynamic test data on modern aircraft tires and this lack has hampered efforts to validate the existing tire models. For taxi performance, the inability to predict and model the gas/fluid mixing was the most serious drawback of the strut model. It was noted that after only a few cycles of strut motion, there were changes in the strut model parameters. Again, as in the case of the tire, there seems to be a lack of reliable experimental data available on the strut performance alone, which could be used to help validate the numerical model.

(10) What criteria are used in evaluating the dynamic performance of the gear system? Can you predict failure of the gear system due to dynamic loadings?

The analytic model of the gear system can be evaluated by comparison of the numerical simulations with drop test or taxi data if such data are available. This usually involves the direct comparison of acceleration time histories or, in some cases, peak loads. Once the gear parameter have been selected so that these comparisons can be achieved, there is reasonable confidence that the numerical simulations can be used to estimate both fatigue and peak loads. Reliance on the simulations has reached the level where there is enough confidence to design metering pins for impact conditions without the aid of drop tests.

The survey responses were brief and there are obviously many other details related to this area which were not included. Fortunately, there did exist a concensus both with regard to the "state of the art" and areas for future concern.

SECTION III

NUMERICAL SIMULATION AND PARAMETER SENSITIVITY STUDY

In conjunction with the review of the current modeling techniques, a digital numerical simulation of the response of a simplified oleo-pneumatic landing gear was performed. The purpose of the simulation was to evaluate the sensitivity of the gear response to variations in parameters used to describe the gear. The simulations were performed by numerically integrating the nonlinear differential equations of motion of the aircraft/gear/tire system. The numerical model was "taxied" over discrete runway disturbances and the time history of the system response was monitored.

It is important to realize that very few definite conclusions can be drawn from this type of simulation study. The solution of the nonlinear differential equations resulting from each sequence of integrations is a particular solution and the generalization of the results from such a solution is difficult. The particular solution is dependent upon the parameters used to describe the system, initial conditions and forcing function. The comparison of two different simulations is limited to the comparison of the time histories of the state variables such as position, velocity and acceleration. Since general solutions are not developed, the sensitivity of the solutions to the model parameters can only be determined by conducting numerous simulations in which single parameters are varied. This is the type of study described in this section.

1. ANALYTIC MODEL (GEAR AND TIRE)

A single chambered, cantilevered oleo-pneumatic strut was modeled for the simulation. A schematic of this type of strut is shown in Figure 1. The aircraft was assumed to be rigid and represented as a single lumped mass attached to the top of the strut. The lower strut, wheel and tire were lumped into another mass so that the complete dynamic model was the two-degree-of-freedom lumped mass model shown in Figure 2. The internal gear forces were represented using the nonlinear pneumatic spring and velocity squared damping discussed in the previous section. The pneumatic force was represented as:

$$F_{pn} = \frac{p_o\ (V_o)^n\ A_{pn}}{(V_o - s \cdot A_{pn})^n} \qquad \text{(III-1)}$$

where F_{pn} = pneumatic force

p_o = fully extended strut initial preload pressure,

V_o = fully extended strut gas volume,

A_{pn} = effective pneumatic area,

s = gear stroke (positive as measured in compression from full extension)

and n = polytropic gas constant.

The baseline parameters used throughout this study were those used in the analysis and tests of Reference 23 and are included in Table I. The gas volume is determined by the strut geometry and the amount of hydraulic fluid, and was maintained constant for a given simulation. The preload pressure could be varied and, in practice, is occasionally used to modify taxi performance. The static load stroke curve for the values of the polytropic gas constant (1.0, 1.1, 1.4) are shown in Figure 3. These are given for a preload pressure of 45 psi and illustrate the nonlinear character of the pneumatic "spring." For this particular gear, the static equilibrium condition corresponded to a constant pneumatic force of 2410 lb (i.e., the aircraft weight supported by the strut). This force corresponds, for an isothermal compression (n = 1.0) and p_o = 45 psi, to a stroke of 0.520 ft. The total allowable stroke was 0.615 ft so that, with this preload pressure, there was less than 0.1 ft of compressive stroke remaining before the gear would bottom. As the maximum stroke is approached, or as the denominator of equation (III-1) approaches zero, the pneumatic forces become unrealistically large. For the very small time steps used during the integration process, these very large forces preclude the possibility of strut bottoming for the cases studied.

The initial conditions for the simulations were determined by assuming an isothermal (n = 1) compression of the gas from the fully extended position to a pressure required to support the weight of the upper mass. This established an equilibrium strut pressure and stroke which were used as starting values for the integration process. These values depended only on the preload pressure for this simulation.

Inspection of Figure 3 indicates the nonlinear character of the air spring. There is a hardening of the spring with increased stroke (i.e., increased slope) and a softening with decreased stroke. There are, obviously, significant variations in slope ($0 \rightarrow \infty$) over the range of the stroke. Increased preload pressure results in a shift in the curve, upwards and to the left. The increased preload pressure effectively alters the slope of the load stroke curve in the region of the static load. If the maximum static pneumatic force is used as an upper bound on gear performance (as is often done), an increased preload pressure may actually reduce the amount of stroke available during taxi. Two different preload pressures were used during this study, 45 and 90 psi.

The internal hydraulic force was represented using the model outlined in Reference 23. The hydraulic force opposes and is in phase with the stroke rate; it can be expressed as:

$$F_{hy} = \frac{\rho \, A_{hy}^3 \, \dot{s}|\dot{s}|}{2 \, C_d^2 \, A_{or}^2} \qquad \text{(III-2)}$$

where F_{hy} = hydraulic force

ρ = density of the hydraulic fluid

A_{hy} = effective hydraulic area

C_d = orifice discharge coefficient

A_{or} = open orifice area

and $\dot{s}$ = stroke rate (positive for compressing strut)

Here, the discharge coefficient, C_d, was the parameter of interest; the others are fixed by the geometry of the gear. The orifice area can be a function of stroke position, depending upon the geometry of the metering pin. Obviously, each metering pin geometry will then result in a different gear performance during taxi. To eliminate this dependence upon pin geometry, a constant net orifice area was used for this simulation, which would be the case for either a constant area metering pin or no pin. It would be difficult to draw any conclusions with regard to the influence of metering pin geometry without conducting many costly and time-consuming simulations, although it is worth noting that it would be much more efficient than conducting the same number of actual taxi tests with numerous metering pins.

Discharge coefficients of 0.8 to 1.25 have been used in other documented simulations,[3] and some experimental data[21] relative to steady orifice flows indicate values ranging from 0.60 to 0.95. These values depend upon orifice geometry (sharp versus rounded edges) and orifice Reynolds number. Values in the range of 0.80 to 0.90 seem rather common, and that was the range used during these simulations.

Another internal strut force usually found in many simulations is bearing friction. For the simple, vertical cantilevered strut used in this study, there was no mechanism for generating bearing normal forces and, thus, bearing friction. It was decided that inclusion of a bearing friction for this simple case would be unappropriate but this is not to imply that bearing friction is not an important or critical component of the net internal strut force. To represent the bearing forces adequately, a more detailed definition of the geometry of the gear would have been required as well as the inclusion of drag loads or strut deformation.

The tire is the last of the force transmitting mechanisms to be discussed. As seen from the results of the study, the selection of the tire model is particularly critical. To keep the level of detail of the tire model consistent with that used in other studies, a point contact spring model was selected. Again, variations in the model similar to that in Reference 23 were chosen. Based on the dynamic force deflection characteristics of a "typical" (but obviously outdated) Type I, smooth contour tire,[23] three tire models were developed, e.g., linear, linear-segmented and nonlinear, whose force-deflection curves are shown in Figure 4. The models did not account for hysteresis, although this could also be an important phenomenon during taxi performance. There was also no viscous damping associated with the tire spring or dependence of stiffness characteristics on the tire rotational speed. The tire force was zero if the height of the unsprung mass above the runway surface exceeded the value it would have for an undeformed tire. The tire was allowed to bounce.

The linear model is the easiest to use but probably the most unrealistic. Due to the nonlinear, large amplitude deformation characteristics of an actual aircraft tire, the applicability of the linear model is quite limited. The nonlinear models attempt to account for the large increase in tire stiffness associated with tire bottoming. The details of the tire models and associated parameters are given in Table II.

A set of two, second-order, nonlinear ordinary differential equations were developed to describe the dynamic response of the aircraft/gear system. These equations were reduced to four first-order differential equations and numerical solutions developed. These solutions were the transient response from the condition of static equilibrium due to a base motion at the point of contact of the tire with the ground.

2. RUNWAY MODEL

Although numerous methods exist for modeling both rigid and nonrigid runway surface, two simple "bump" models were used for this study - the series of 1-cos bumps and a single repair mat, as shown in Figure 5. The surface was assumed to be rigid with the simulation initiated at the start of each disturbance. For the 1-cos bumps, both frequency and amplitude could be varied. The frequency is related to the wavelength of the bump and the aircraft velocity. For both disturbances, the forward velocity of the aircraft was constant as it passed over the disturbance.

3. NUMERICAL SIMULATION PROCEDURE

The system of four, coupled, first-order ordinary differential equations of motion for the two-degree-of-freedom dynamic system was numerically integrated using a fourth order Runge-Kutta integration scheme. All solutions were generated using the University of Notre Dame's IBM 370/168 computer. A single code was written, incorporating all the tire and disturbance models. Output was in the form of both printer and ink plots of time histories of position, velocity and acceleration of both upper and lower masses. Output also included time histories of stroke, hydraulic, pneumatic and tire forces. Peak value of velocities and accelerations were also recorded. Each simulation yields a large amount of data which can create difficulties when attempting to interpret the results of a given simulation. For presentation within this report, many of the results are summarized in terms of peak total acceleration (or g's).

The time increment for the integrations was chosen to be an order of magnitude smaller than that required to produce variations in the numerical solution on the order of 1%. A Δt of 0.002 sec was used for a series of 1-cos bumps at a frequency of 1.0 Hz, a Δt = 0.0002 sec for a frequency of 10.0 Hz, and a Δt = 0.0001 sec for a frequency of 50 Hz. These time increments were selected so that the high frequency transients, present as the disturbances were first encountered, were adequately represented.

4. RESULTS

The simulation was conducted to determine the influence on the response of a particular gear system to variations in a number of characteristic parameters. Those parameters of interest were the polytropic constant, orifice discharge coefficient, preload static gas pressure and the tire stiffness. The simulations were conducted using both a series of 1-cos bumps and a simple patch profile.

To illustrate the type of response characteristic of this two-degree-of-freedom system, a sequence of time histories for aircraft and wheel vertical positions, gear stroke and gear forces are presented in Figures 6-12. Each of these simulations was conducted using the linear segmented tire model and the "baseline" values for n (1.0) and C_d (0.90). Figures 6-10 illustrate the influence of frequency and amplitude of the (1-cos) disturbance on the system response.

A frequency of 1 Hz, Figures 6 and 7, would be considered a relatively low frequency when considering taxi response to runway damage. Lower frequencies could be encountered for very large wavelength disturbances but, since most aircraft structural frequencies would exceed a value of 1 Hz, it was considered as a lower bound on the frequency range. At this frequency, both the upper and lower mass follow the ground profile with a very slight phase shift between their positions and a small amplitude overshoot. Gear stroking occurs with its amplitude approximately 1/3 of the bump height. Tire deformation "absorbs" the other 2/3 of the bump. The stroking rate is relatively low so that, particularly for the small bump (0.25), the hydraulic forces are quite small. There are higher frequency fluctuations present in the stroke and each of the forces. The results are presented for five cycles of "bump" input and it is apparent that the transients are still present and a steady state solution has not been achieved. For both small and large amplitude bumps the tire remained in contact with the surface and the strut did not approach a bottoming condition.

In the next sequence of simulations, Figures 8 and 9, the frequency of bump encounter has been increased to 10 Hz. The character of the response has changed dramatically. The upper mass responds at a predominant frequency of approximately 1.7 Hz for the 0.25 ft amplitude bump. The lower mass responds at the lower frequency with the bump frequency of 10 Hz superimposed. The amplitude of the high frequency stroking is approximately ho/3 with the tire deformation absorbing the remainder of the bump. Due to the increased stroking velocity, the hydraulic forces have increased by two orders of magnitude over the previous case. This brings about the pronounced damping of the transient response. For the small amplitude bump, the tire leaves the ground during a few of the dips near the beginning of the response. For the large amplitude bump, the tire "bounces" dramatically upon impact with the first bump. This brings about a strong peak in the pneumatic force, which would be characteristic of strut bottoming. The magnitude of the hydraulic forces is comparable for both bump amplitudes. In both cases, the steady state response is governed by the ability of the tire to accept the deformation caused by the surface disturbance.

For the final case with the periodic bump disturbance, Figure 10, an encounter frequency of 50 Hz was used. Only the 0.25 ft amplitude case is illustrated since the larger amplitude disturbance created a "bounce" of such amplitude that subsequent results appeared physically unrealistic. For the case shown, both the upper and lower masses respond in phase with a frequency of approximately 1.7 Hz. The lower mass has a low amplitude, high frequency component superimposed upon the lower frequency response. Gear stroking occurs at the lower frequency with a small amplitude, high frequency component superimposed. The stroking amplitudes are small but the rates large enough to bring about significant hydraulic damping forces. The tire, again, must absorb a majority of the deformation and, in the steady state, the gear is simply "bouncing" from crest to crest on the periodic ground disturbance.

The response of the system to the patch ground profile is illustrated in Figures 11 and 12. In each case, the baseline gear parameters (C_d = 0.90, n = 1.0) and the linear segmented tire model are used. At an aircraft encounter speed of 25 ft/sec, the system undergoes a damped periodic response due to the disturbance created by entering and leaving the ramp. The two masses respond in phase. There is significant srroke movement and tire deformation, with peak forces occurring as the tire leaves the ramp. At the highest encounter speed of 100 ft/sec, Figure 12, the aircraft responds at its "natural" frequency of 1.7 Hz and, obviously, spends far less time on the patch. For this case, the peak gear forces occur as the system encounters the patch; the system "flies" off the end of the patch and the tire loses contact with the ground as it leaves the patch. For a larger amplitude patch (h_o = 0.50 ft.) , the system bounces as it encounters the patch, as in the 1-cos bump cases.

The remainder of the results will present the sensitivity of system response to changes in various gear parameters. Figures 13 and 14 illustrate the influence of variation of polytropic constant, discharge coefficient and preload pressure. The peak accelerations which are proportional to the peak total forces are used to evaluate the influence of their parameters. All the results presented in these two figures were developed using the linear tire model and other parameters indicated on the figures and the (1-cos) surface roughness.

Figure 13 illustrates the influence of both the polytropic constant and the preload strut pressure. Although there was not a strong variation in peak acceleration with change in n over the range of 1.0 to 1.4, at low frequencies the changes in peak load over this range were on the order of 10%. It is interesting to note that at low preload pressure (45 psi), there is an increase in peak acceleration with an increase in n and, at the high pressures (90 psi), there is a decrease in peak acceleration with increased n. This occurs for both the "aircraft" and the "wheel." At the higher frequencies the influence of the gas constant was reduced but, particularly at 10 Hz, the preload pressure had a significant influence. The increased preload pressure resulted in nearly a 25% decrease in peak acceleration, and this occurred for all values of n over the range considered. At the highest frequency considered (50 Hz) the influence of the preload pressure was not so dramatic, although the increased pressure still resulted in a decreased peak acceleration.

The influence of the orifice coefficient is illustrated in Figure 14. The coefficient was varied over a range of 0.80 to 0.90 with other parameters fixed, as indicated on the figure. Again, the linear tire model was used with the (1-cos) ground roughness profile. The influence of C_d on the peak acceleration for the range of C_d values considered varied from a very slight increase at the low frequency (1 Hz) to changes on the order of 10% at the higher frequencies. It is interesting to note the relatively complex dependency on C_d. For a 10 Hz encounter frequency, the peak aircraft acceleration increases at a preload pressure of 45 psi for increased C_d and it decreases for increased C_d at the higher preload pressure of 90 psi. At a 50 Hz encounter frequency, the peak aircraft acceleration decreased with increased C_d at both preload pressures. The discharge coefficient had a greater influence on the peak wheel accelerations than it had on the peak aircraft accelerations.

The final study conducted examined the impact of the tire model on the simulation results. For each of these cases $C_d = 0.90$ and $n = 1.0$, and Figure 15 presents the results for each tire model at various disturbance heights for the (1-cos) roughness with an encounter frequency of 10 Hz. As would be expected for low amplitudes (i.e., < 0.2 ft), there is little difference between the tire models since tire deflections are small. For higher amplitudes, the tire model has a significant effect. As the tire nears a bottoming condition, only the linear segmented model accounts for the drastic increase in stiffness. This illustrates that if large tire deformations are involved, the tire model and characteristic parameters are extremely important. It is reasonable to assume that the segmented tire model provides the best description of the tire characteristics and it consistently yielded higher peak accelerations for both aircraft and wheel. For the results shown in Figure 16, a fixed simple patch ground profile was used with a height of 0.25 ft. The aircraft encountered the patch at a fixed speed. Again, the greatest aircraft accelerations were predicted using the linear segmented model, with the linear model approximately 20% lower. The wheel response results were more complex, with the linear model prediction greater at the low speed and a reversed trend at the higher speeds. The sensitivity of the individual tire models to variations in their characteristic parameters were not evaluated.

In interpreting these results, it is imperative to realize that they reflect the response of a particular gear system to a very limited number of operating conditions. This indicates the drawback mentioned previously in this type of numerical solution. Although certain trends may be apparent from these results, it would be foolish to attempt to draw any general conclusions with regard to parameter sensitivity. The one conclusion that can be drawn is that there are certain conditions such as roughness amplitude, encounter frequency and preload pressure where the results of the simulation can depend strongly on the choice of C_d, n or the tire model. In cases with more involved gear geometries, more complex surface roughness distributions and a wider range of operating conditions, the choice of the best set of gear parameters for a given system may not be at all obvious.

SECTION IV

PARAMETER IDENTIFICATION

As discussed in the previous section, the accurate prediction of parameters to describe the dynamic characteristics of the gear and tire system are required to adequately predict the dynamic system response. Parameters such as discharge coefficient, polytropic constant and tire stiffness can either be determined from analytic techniques or by experimentation. Due to the complex nature of the phenomena involved, there appear to be no reliable analytic techniques available for predicting these parameters. As mentioned, these parameters are usually determined experimentally. Static load deformation data, usually developed by the manufacturers, are used for tire models. Either experience or the correlation of numerical simulations with drop test data is used to estimate most gear parameters. The following section outlines a systematic technique for determining certain gear and tire parameters which could be applied to help automate the prediction of gear and tire parameters.

1. BACKGROUND

The problem of system parameter identification for dynamic systems is encountered in a number of areas. In a system where the dynamic response (i.e., positions, velocities and accelerations) can be accurately measured, these data can often be used with existing analytic models, either in the form of differential equations of motion or their solutions, to help describe various system parameters. If the differential equations can be solved analytically, these solutions can often be matched to the experimental data and various system parameters can be determined.[29] If an analytic solution is not possible, then either analog or digital numerical solutions to the differential equation can often be obtained. Then, by "adjusting" the parameters in the numerical model, these solutions can be fitted to appropriate experimental data. The fitting procedure can either be based on skill and experience of the analyst or there are systematic techniques which can be employed.[30-33] Reference 33 outlines a series of tests in which such a procedure was applied to a mechanical system to determine stiffness and damping parameters. The technique used in that work is referred to as Newtonian iteration. The application of this technique to the landing gear problem will be discussed in the next section.

2. APPLICATION TO GEAR SYSTEM

The Newtonian iteration can be applied to single or multiple degree of freedom dynamic systems. The case considered here is a single-degree-of-freedom system which can be used to determine two of the important landing gear strut parameters, C_d and n. A similar development could be performed for a "tire alone" system, in which tire stiffness and damping characteristics could be obtained. The technique could also be applied to the two-degree-of-freedom strut and tire system, similar to that used in the previous simulations.

Consider the system shown in Figure 17a, in which a rigid mass, m, is attached to the upper strut of a simple, vertical, cantilevered gear. The lower strut is fixed. The system would oscillate if disturbed from equilibrium, as shown in Figure 17b. The equation of motion for this system can be written in terms of the gear stroke, s. Assuming that the intra-gear forces are the hydraulic and pneumatic forces discussed previously, the equation takes the form:

$$m\ddot{s} + \frac{\rho \, A_y^3}{2(A_{or}\, C_d)^2} \, |\dot{s}| \, \dot{s} + p_o \, A_{pn} \left[\frac{V_o}{V_o - s \, A_{pn}} \right]^n - mg = 0 \qquad \text{(IV-1)}$$

If all geometric parameters are fixed, as well as the preload pressure and hydraulic fluid density, the response depends upon C_d, n and the initial conditions or boundary conditions imposed on the system. As stated, the goal is to "fit" a numerical solution of this differential equation to an experimental position time history to determine the unknown parameters C_d and n. The accuracy of the fit can either be assessed in a subjective manner, as is done in some analog simulations, or it can be based on a definite criterion. In the procedure suggested here, a least squares criterion is recommended for determining the accuracy of the fit. By defining the residual as the difference between an experimental data value and the result of the numerical solution at a given instant in time, the optimum "fit" is one which minimizes the sum over all the data points of the residual value squares.

This technique has not been applied to the landing gear problem but it appears as though it would be suitable. Appendix B outlines the steps required for the application to the dynamic system given in equation (IV-1).

If the technique from Appendix B were developed and automated, a sequence of tests could be conducted to verify both the accuracy of the procedure and its suitability for this particular problem. The required experimental data would be stroke or position time histories. These could be acquired using one of two techniques. In some previous applications,[32,33] high speed photographic records of the motion were collected and positions measured manually from a frame-by-frame sequence. An alternative to this somewhat time-consuming method would use position transducers and on-line analog to digital conversion. Such an automated technique would allow for a wide variety of parameter and test conditions. It could be used on either model or actual gear systems.

The procedures outlined above may be suitable for defining specific gear parameters for an actual gear system. The size and complexity of the apparatus required for such a test would depend upon the particular gear system and the type of test conditions to be simulated. Possibly existing drop test rigs could be modified but they may introduce problems related to mechanical friction. For the dynamic system outlined in the Appendix, no friction forces were included in the differential equation. For a vertical cantilevered strut without significant strut deformation, this may be an adequate model. For a non-vertical or levered strut, where large bearing forces could be developed, the method outlined in the Appendix would have to be modified to include their contributions.

To develop this technique as a method of predicting gear parameters, it would be best to use simple model gear systems in which both testing techniques and data acquisition and reduction procedures could be developed under well-controlled conditions. Consider the schematic of a model test facility shown in Figure 18. The system would be composed of five major components.

a) Gear Support System

The mechanical support for the model gear should be a rigid apparatus in which the gear could be mounted vertically. The "gear" should be easily accessible and supported in such a manner that it could be removed without difficulty. The mass supported on the upper chamber should be variable and capable of representing the extreme operating limits (i.e., bottoming loads, etc.) for the gear. This mass should also be supported on low friction bearings to help minimize the friction influence on gear response. A mechanical or hydraulic "impulse" generator should be part of the support system, to be used to provide either upward or downward initial velocities from either equilibrium or non-equilibrium initial stroke conditions.

b) Model Gear

For initial testing, the simpler the gear the better. A single chamber gear, with no metering pin or snubbers, would provide a basic design. An actual gear could be used but two modifications should be considered. The first would be the ability to use various orifice sizes and shapes (rounded, sharp-edged, etc.). Also, if the main chambers could be constructed of a transparent material, some of the unsteady fluid characteristics such as foaming or bubbling could be observed. Fabricated in this way, certain actual gear parameters such as pressures would have to be reduced below "normal" operating limits but, possibly, some important aspects of the "physics" of the unsteady fluid dynamic phenomena within the gear could also be determined.

c) Instrumentation

Together with the position data which could be collected using a position transducer, other data should be collected during the free vibration tests. The gear should be equipped with both upper and lower pressure and temperature transducers. Either single or multiple transducers could be used to monitor the variation in these parameters during the motion of the gear strut. These data could be used to evaluate the applicability of the analytic models used to predict the intra-gear forces.

d) Data Acquisition

The analog output from the position, pressure and temperature transducers should be collected on-line and converted to digital form for storage, display and subsequent analysis. With the current availability of relatively low cost microcomputers to perform this function, the storage of large amount of analog data should be unnecessary.

e) Data Reduction

This part of the system would involve the software required to perform the numerical fitting procedure. If interfaced directly with the data acquisition system, the results could be available for a near "real" time display. The actual fitting procedure is not particularly time-consuming for a single-degree-of-freedom system when only a few cycles of motion need be analyzed.

Although this system is outlined in only a general fashion, it does indicate that the development of such a system, particularly for a model gear, should be a reasonable endeavor. The opportunity it presents to help further the understanding of all gear systems should prompt such an effort in the future.

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

The report documents a preliminary review of analytic modeling techniques used to describe the influence of landing gear systems on aircraft taxi dynamics. The review involved a survey of the state of the art of gear modeling for taxi simulations and a preliminary study of critical gear parameters.

The survey of both industry and current published reports indicated that there is a "standard" analytic model used to describe the contribution of the gear and tire to the overall aircraft response in numerical simulation of aircraft landing impact and taxi dynamics. This standard analytic model which is discussed in this report, appears to be common to many taxi simulation procedures, although individual organizations have developed additional details for particular applications. The acceptance of the model does not appear to be due to its accuracy or its suitability over a wide range of operating conditions, but it is primarily due to its ease of application and the lack of a more general analytic model. When applied to situations involving "normal" landing conditions or taxiing over reasonably smooth runway surfaces, this gear model can yield good results. The survey did indicate that for certain situations, the model was inadequate and could lead to significant errors in predicting aircraft response. In cases where unsteady effects exist such as air/oil mixing, significant temperature variation or unsteady orifice flows, the "standard" model fails. There was also strong concern expressed with regard to the adequacy of the simple, point contact tire model which is common in most simulations. There appear to be significant shortcomings in current practice when conventional methods are applied to situations involving aircraft taxi performance over severely damaged or repaired surfaces.

The report also outlines a parameter identification procedure which could be applied to the problem of gear parameter definition. It has been shown to be applicable to other nonlinear mechanical systems and may help in the experimental determination of a number of gear or tire parameters. Although such a technique will not eliminate the need for experience in selecting the various parameters required to describe the characteristics of a given gear system, it could remove some of the arbitrary judgements from that decision. It could also be useful in evaluating the dependence of various system parameters on gear operating conditions.

In conclusion, the review has shown that, although in many applications the current methods have provided good results, there are difficulties associated with present gear modeling procedures when applied to dynamic taxi simulations. As gear and aircraft systems are required to operate under more severe conditions, it will be necessary to reevaluate current modeling techniques in order to accurately predict overall system response.

Aircraft weight	=	2410 lb
Tire and wheel weight	=	130 lb
Fully extended strut gas volume	=	0.03546 ft^3
Length of lower strut	=	2.778 ft
Tire radius	=	1.125 ft
Hydraulic fluid density	=	53.5 lb_m/ft^3
Effective pneumatic area (A_{pn})	=	0.05761 ft^2
Effective hydraulic area (A_{hy})	=	0.04708 ft^2
Orifice area (A_o)	=	0.5585×10^{-3} ft^2

Table 1. Simulation Gear Parameters

Linear: $F_t = K\,\delta_t$

$K = 18{,}500$ lb/ft

δ_t = tire deflection (ft)

Linear segmented: $F_t = a\,\delta_t + b$

a) $0 < \delta_t < 0.085$ ft

$a = 6000$ lb/ft $b = 0$

b) $0.085 \text{ ft} < \delta_t < 0.470$ ft

$a = 23{,}400$ lb/ft $b = -1480$ lb

c) $\delta_t > 0.470$ ft

$a = 1.0 \times 10^5$ lb/ft $b = -37.5 \times 10^3$ lb

Exponential: $F_t = a\,\delta_t^{\,b}$

$a = 26{,}520$ lb $b = 1.34$

Table 2. Simulation Tire Models

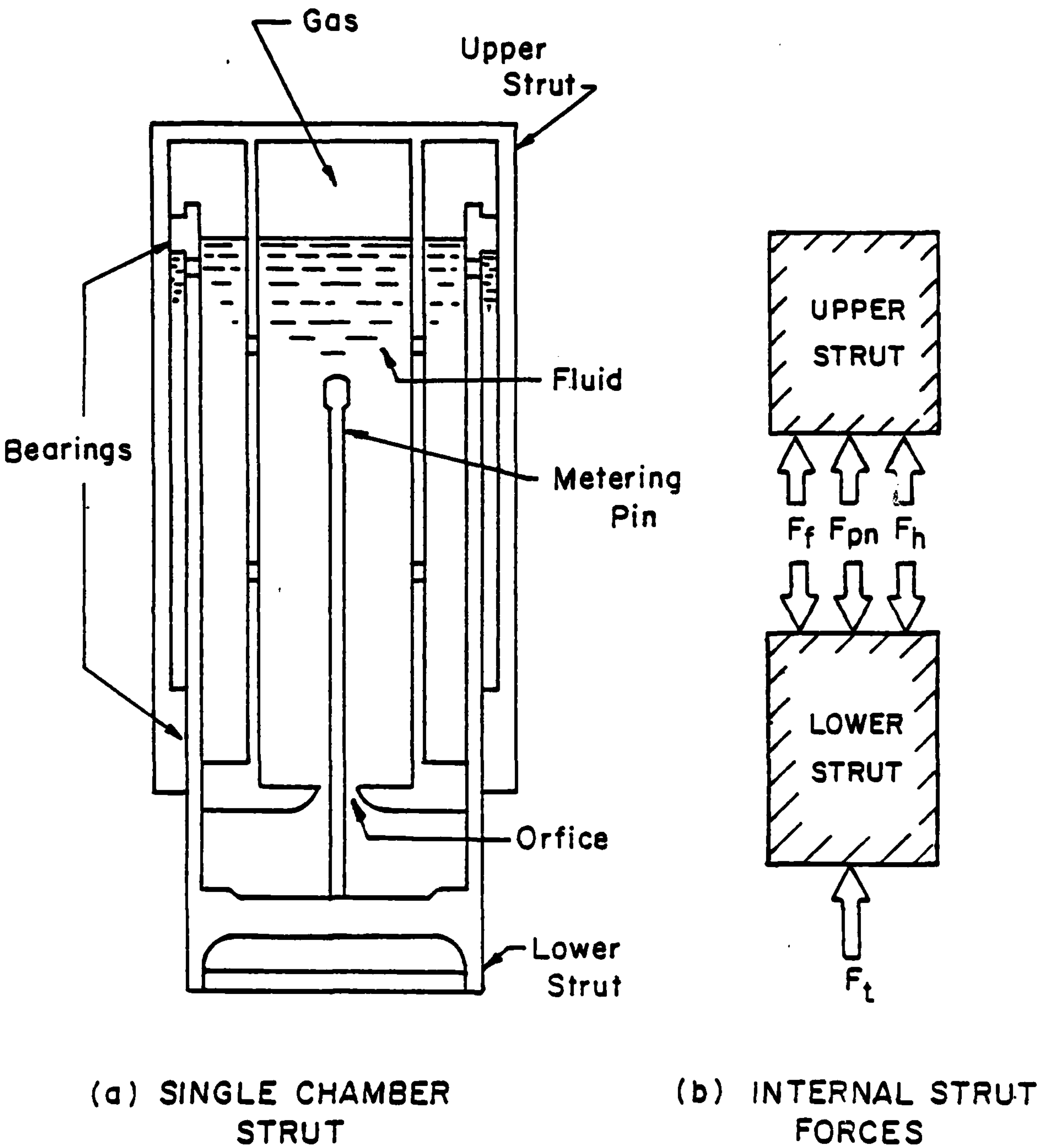
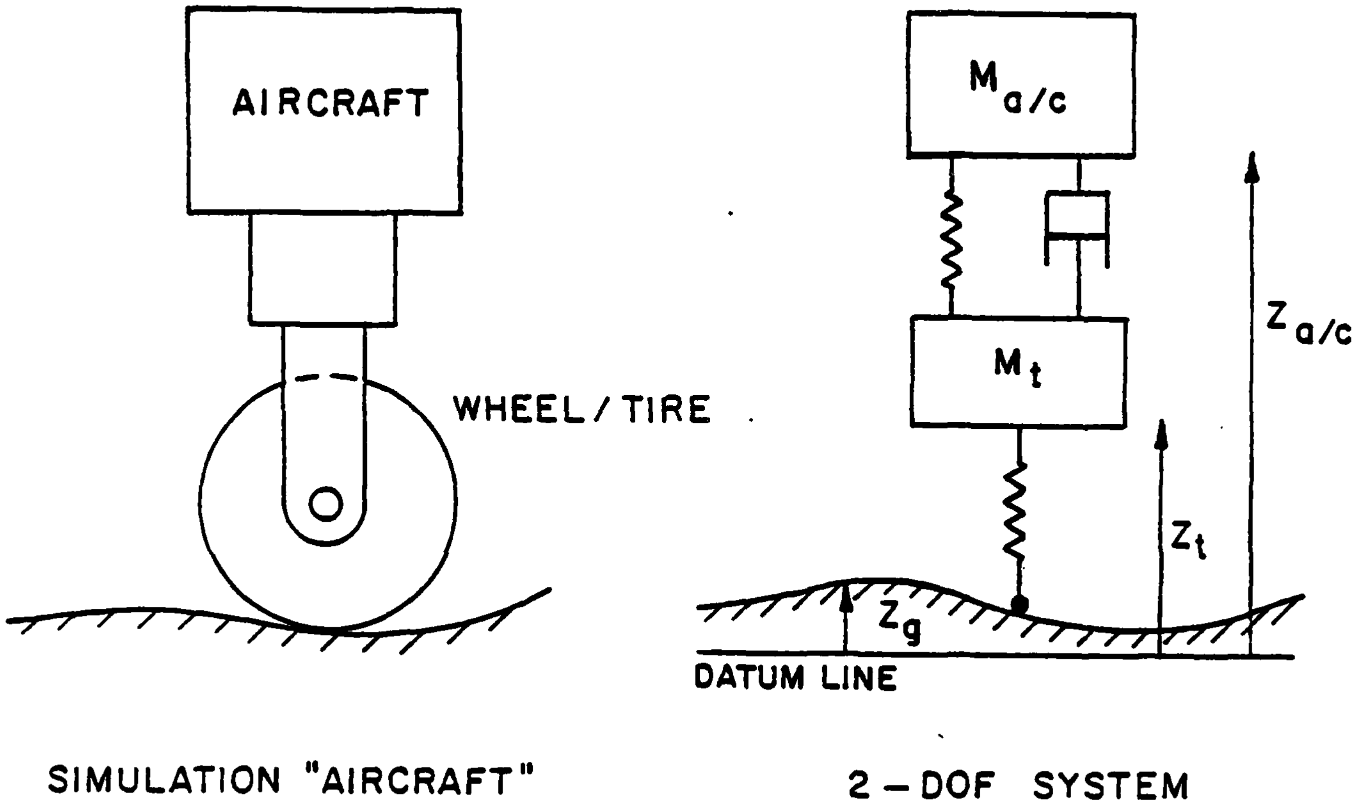


Figure 1. Schematic of Oleo-Pneumatic Strut and Internal Strut Forces



$M_{a/c}$ - Aircraft Mass
M_t - Wheel/Tire Mass
$Z_{a/c}$ - Aircraft Vertical Position
Z_t - Tire Vertical Position
Z_g - Runway Surface Vertical Position

Figure 2. Two-Degree-of-Freedom Simulation Model

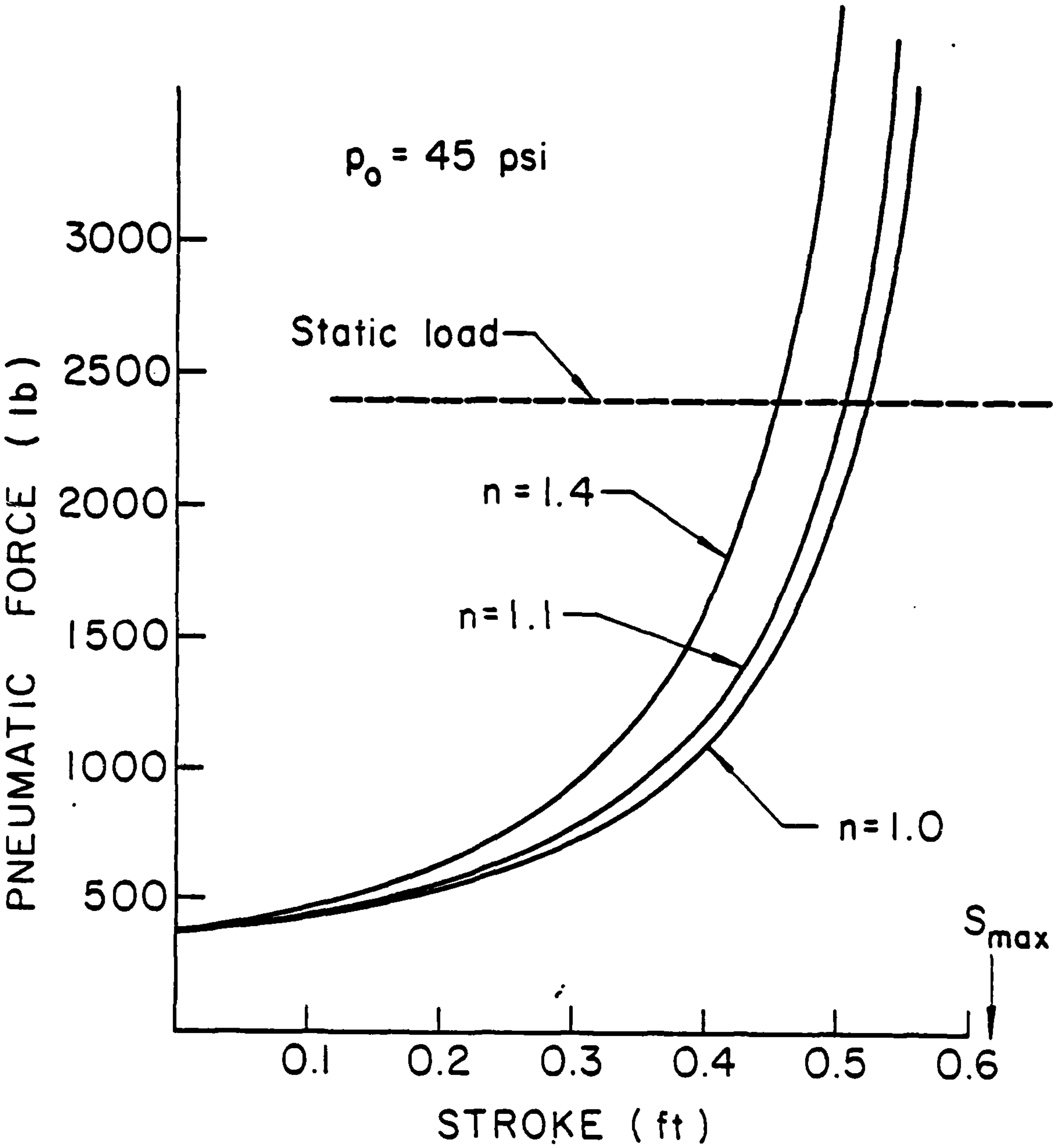


Figure 3. Static Load-Stroke Curves

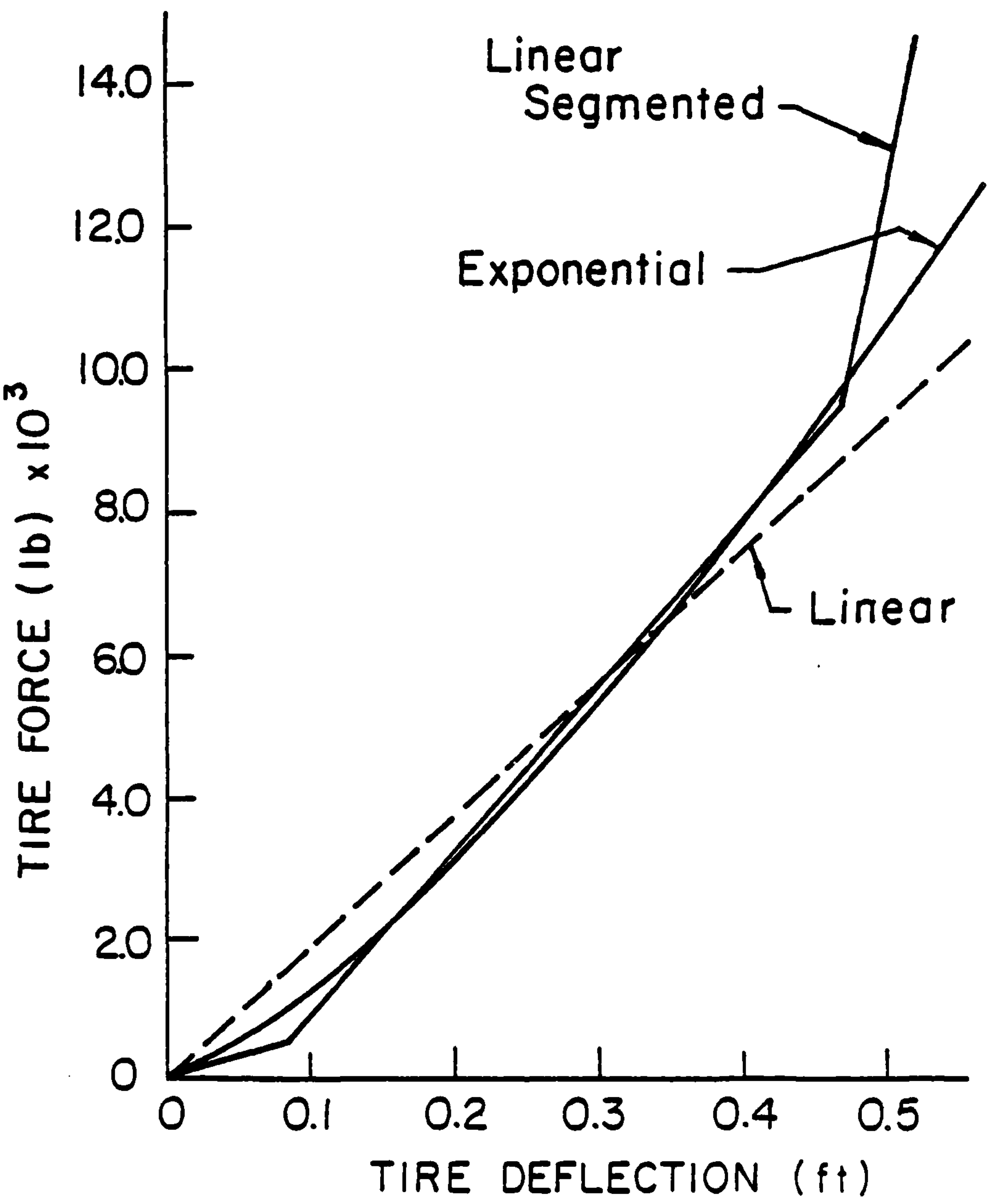


Figure 4. Tire Load-Deflection Curves

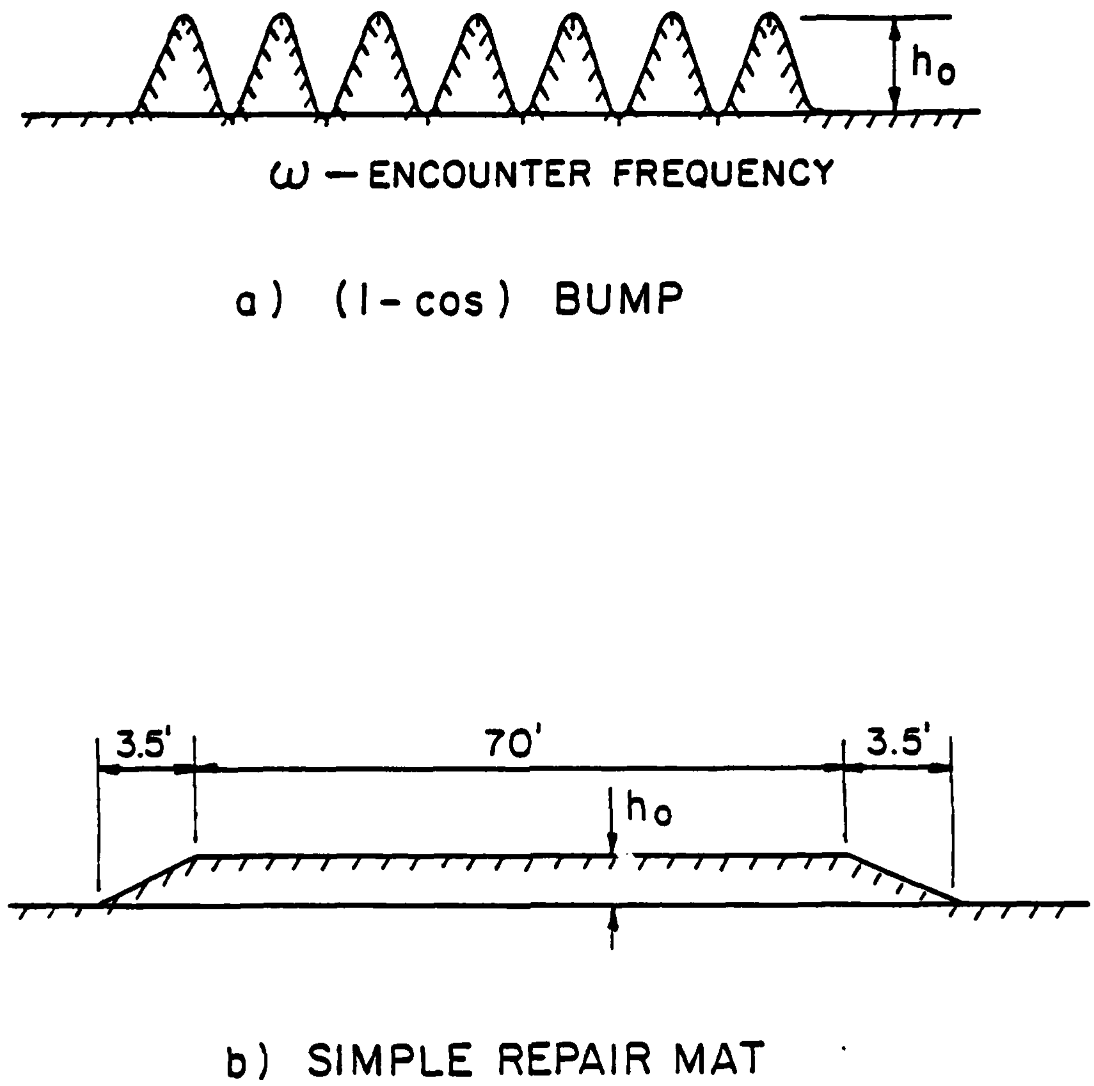


Figure 5. Runway Surface Models

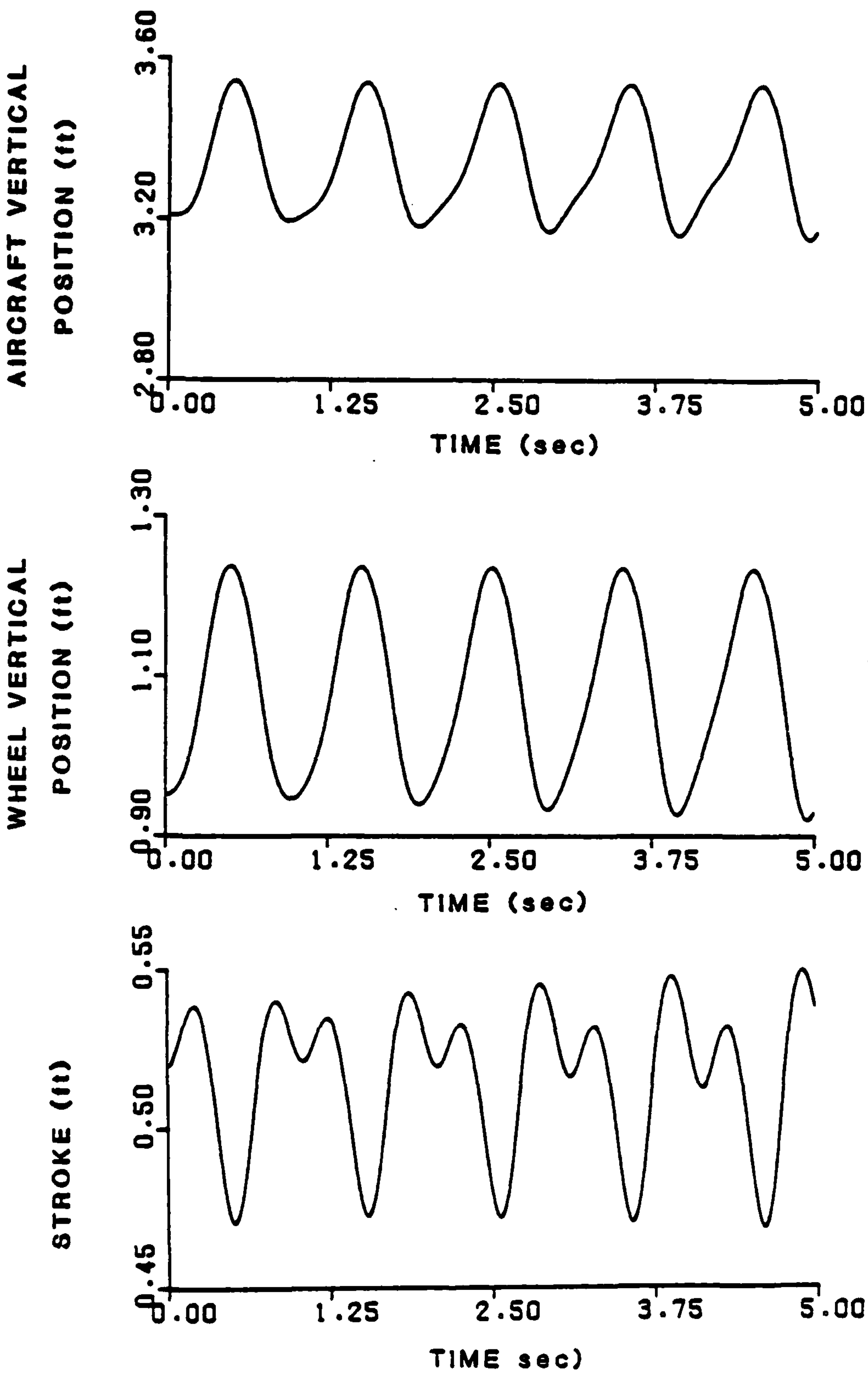


Figure 6a. System Response, 1-cos Profile, Linear Segmented Tire, ω = 1.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

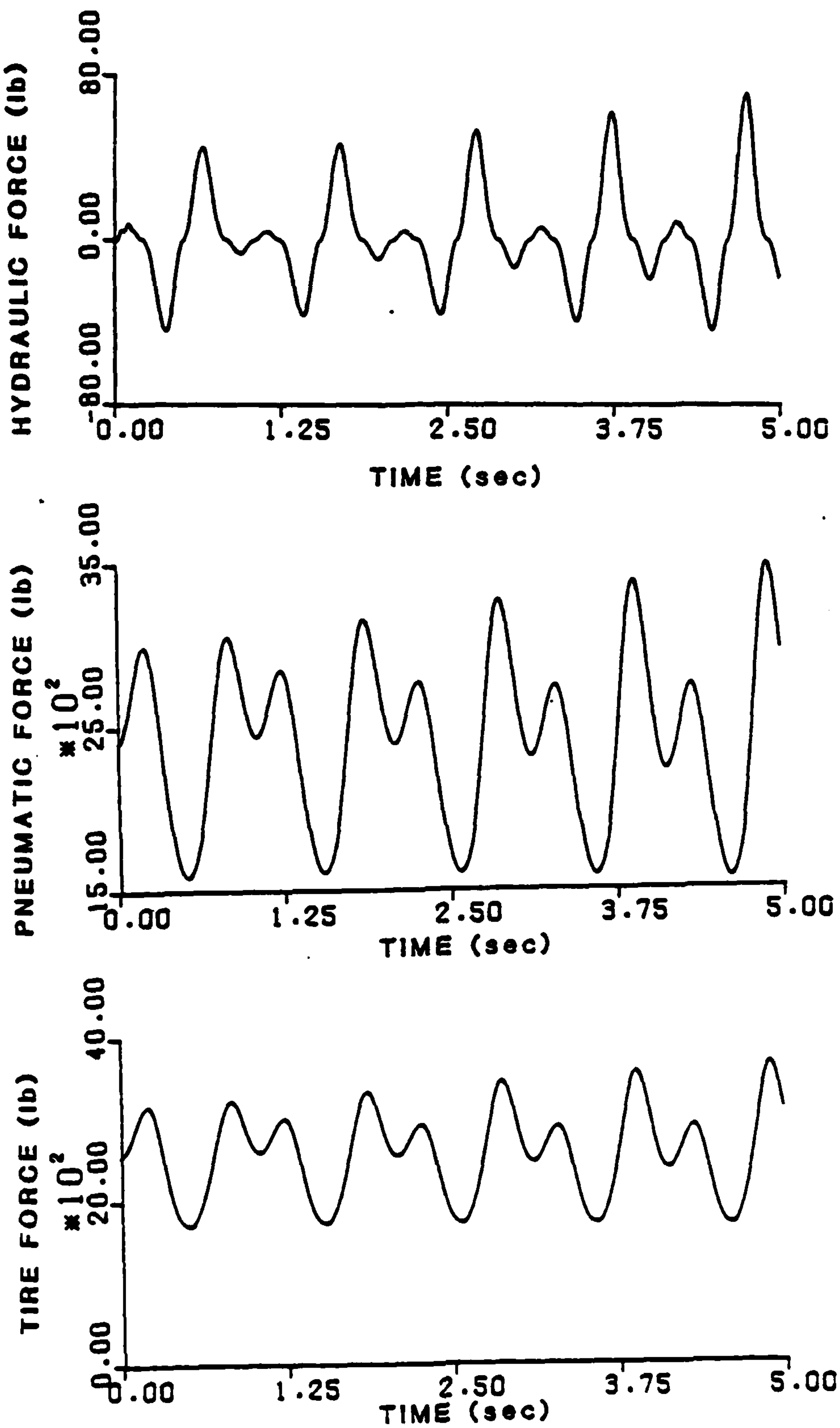


Figure 6b. System Response, 1-cos Profile, Linear Segmented Tire, ω = 1.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

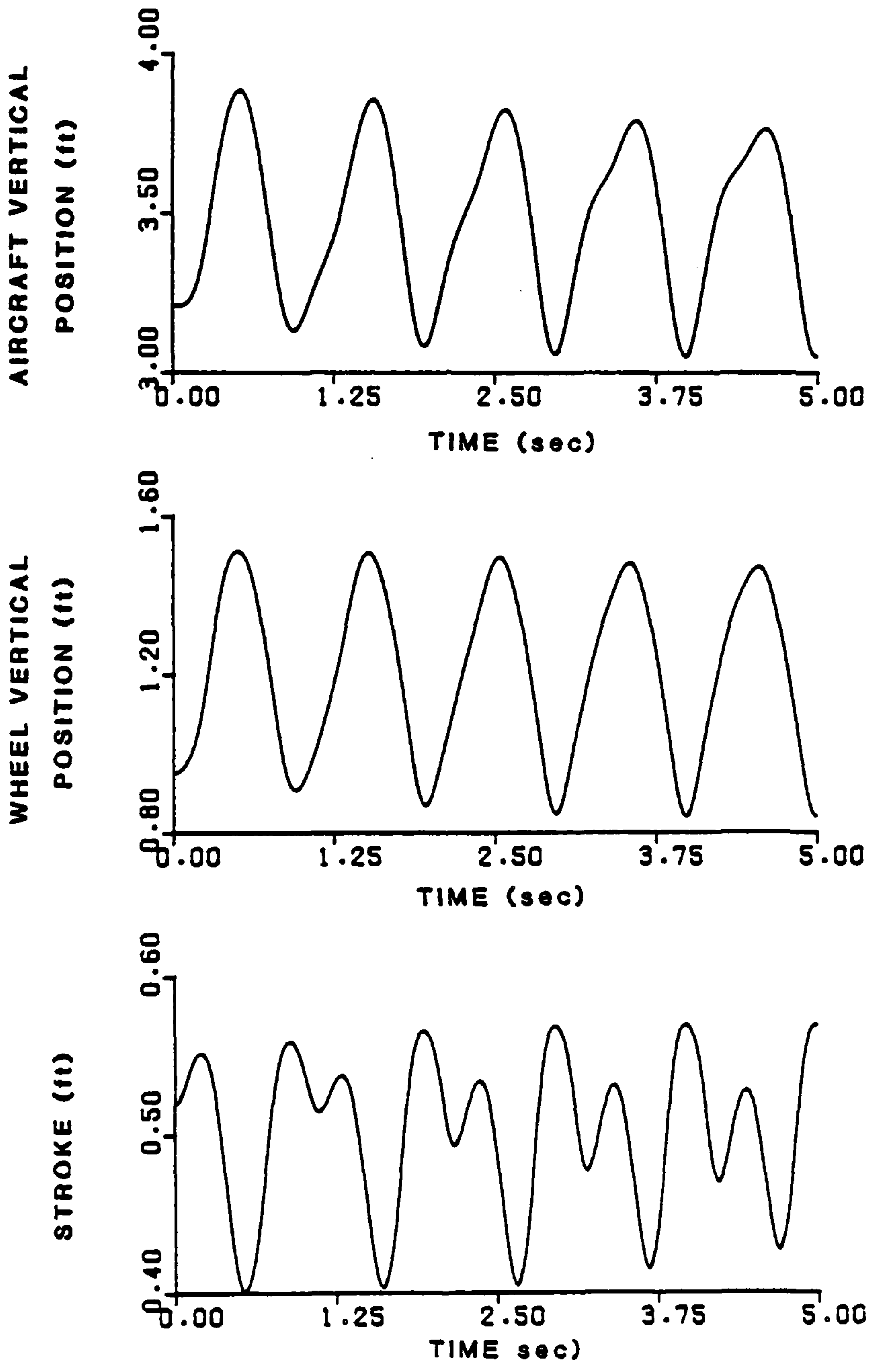


Figure 7a. System Response, 1-cos Profile, Linear Segmented Tire, ω = 1.0 Hz, h_o = 0.50 ft, p_o = 45 psi.

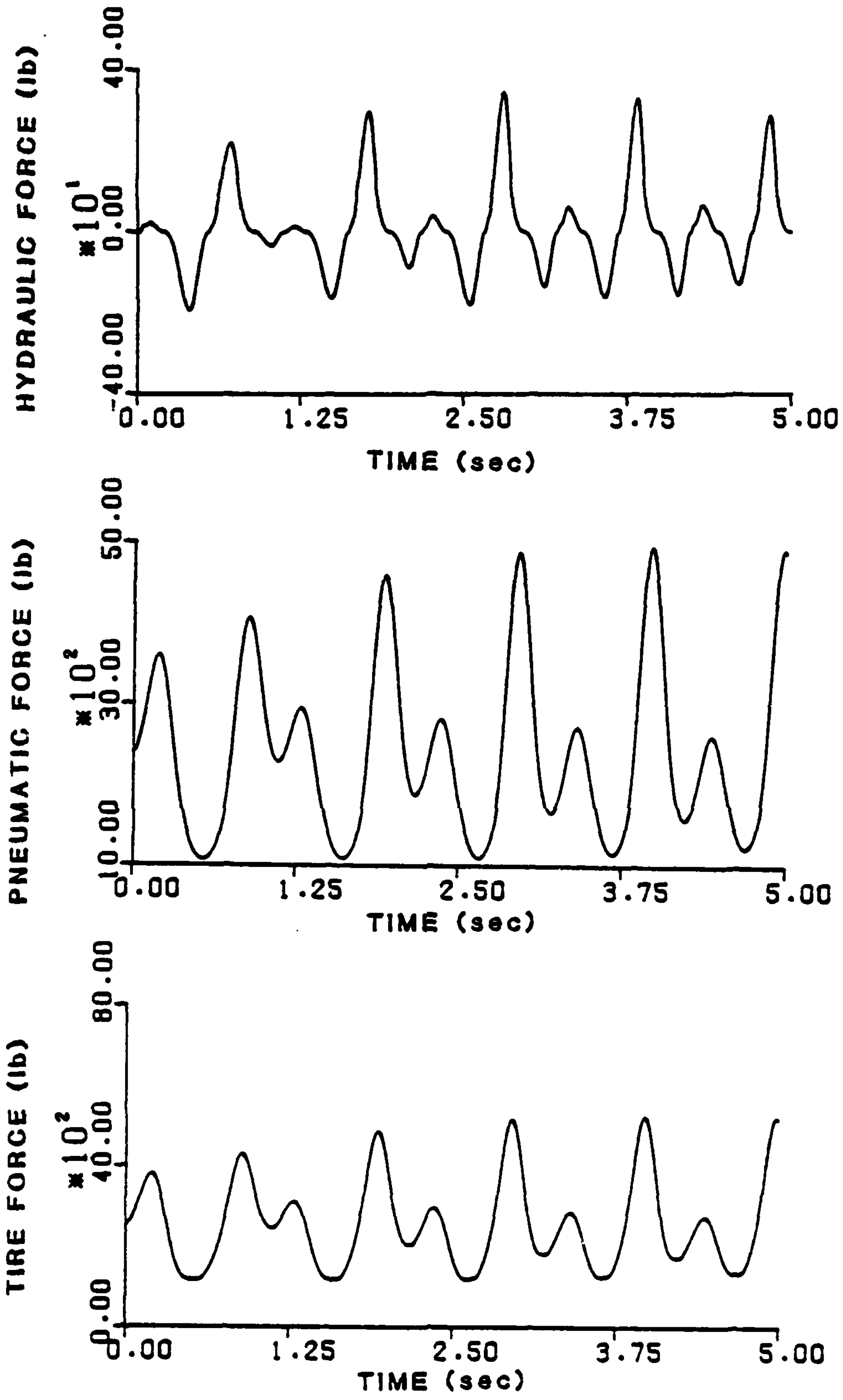


Figure 7b. System Response, 1-cos Profile, Linear Segmented Tire, ω = 1.0 Hz, h_o = 0.50 ft, p_o = 45 psi.

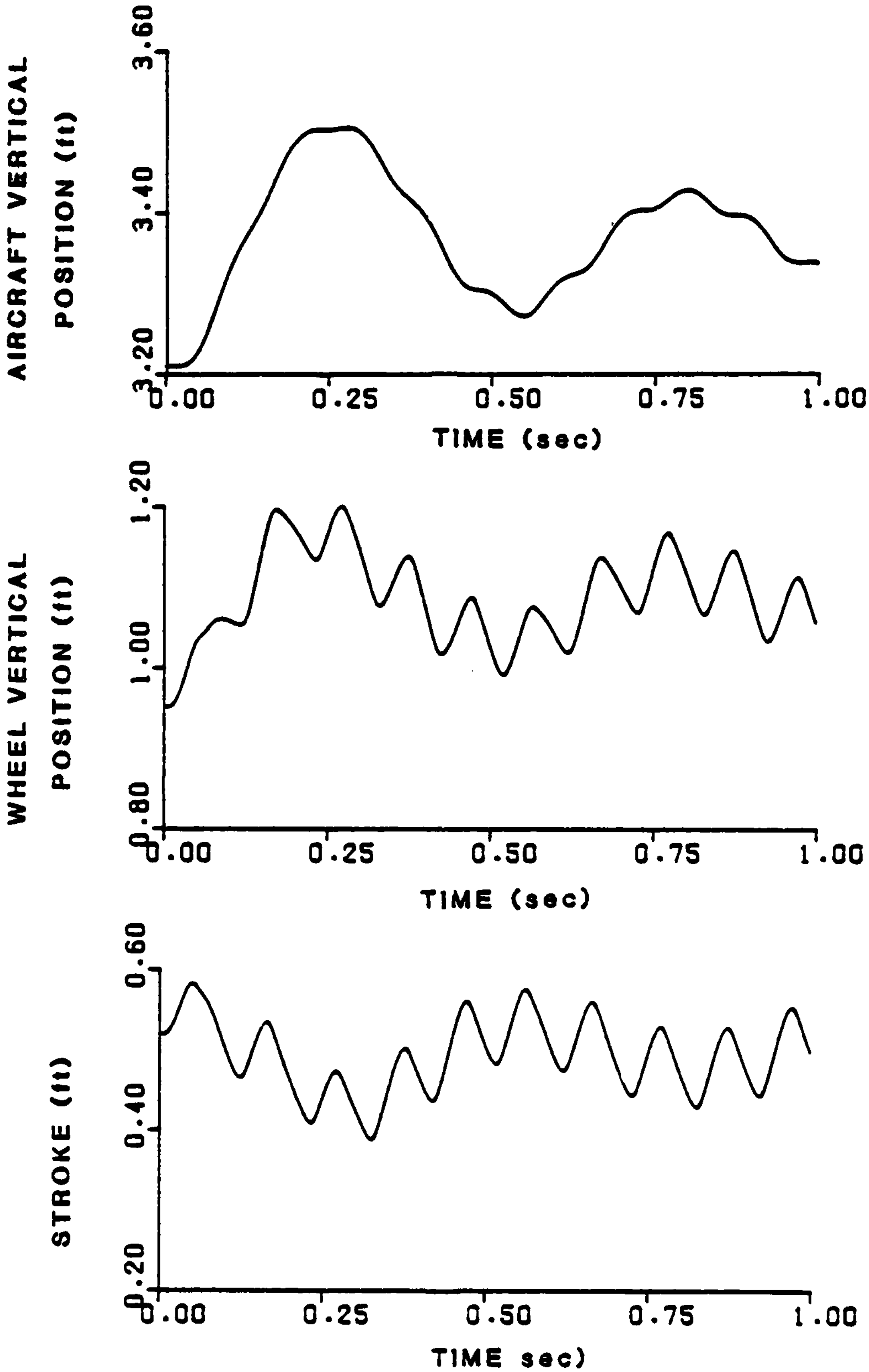


Figure 8a. System Response, 1-cos Profile, Linear Segmented Tire, ω = 10.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

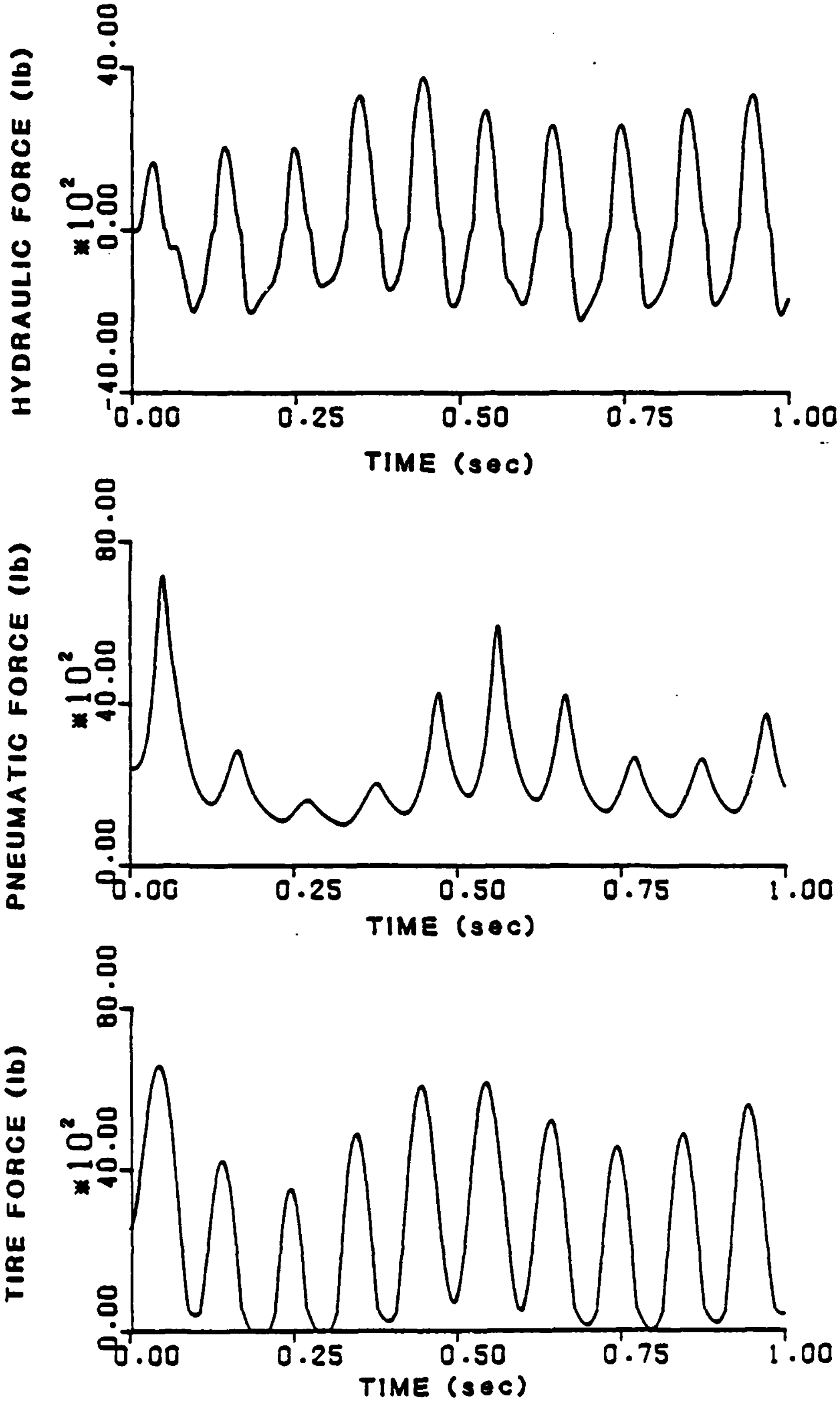


Figure 8b. System Response, 1-cos Profile, Linear Segmented Tire, ω = 10.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

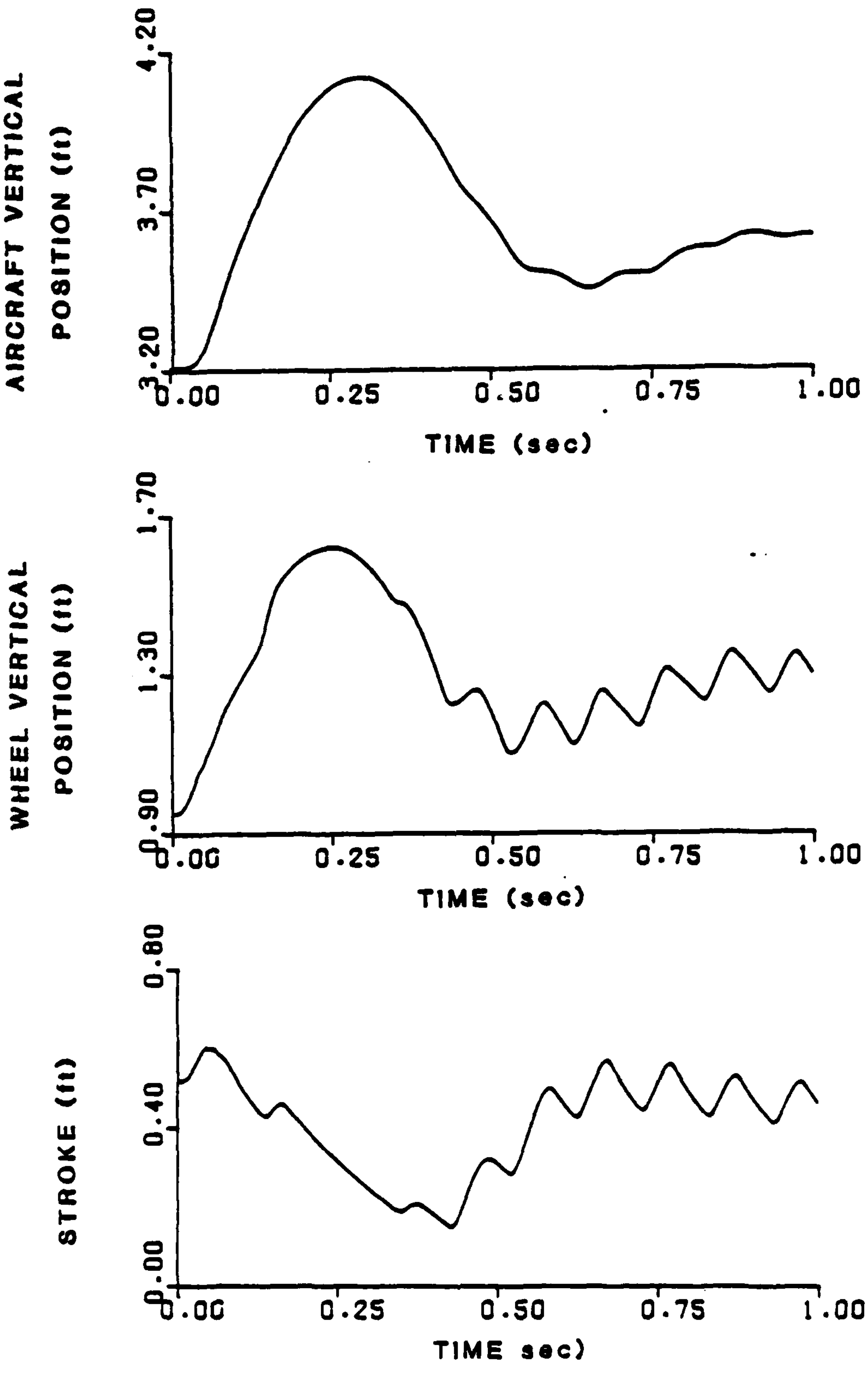


Figure 9a. System Response, 1-cos Profile, Linear Segmented Tire, $\omega = 10.0$ Hz, $h_o = 0.5$ ft, $p_o = 45$ psi.

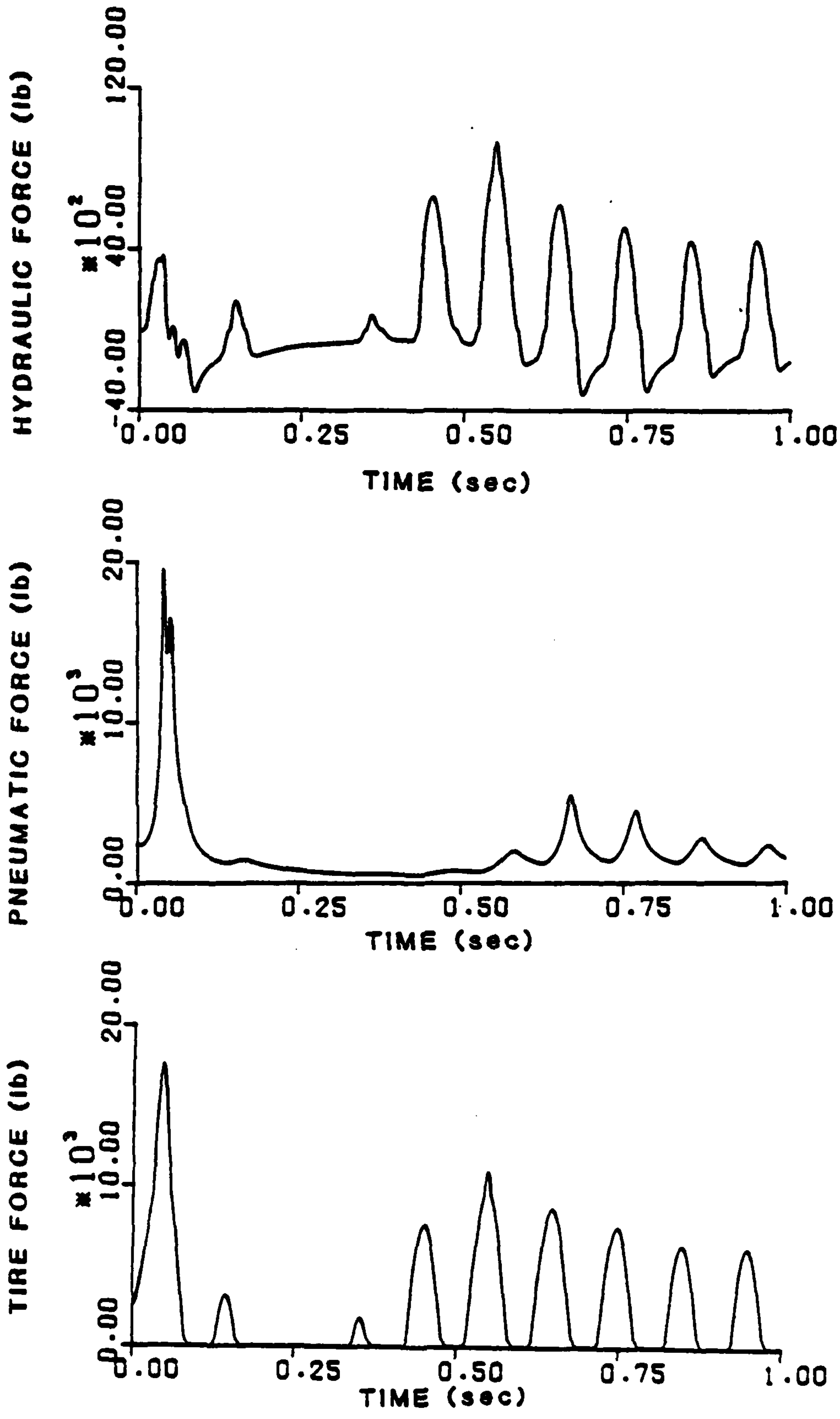


Figure 9b. System Response, 1-cos Profile, Linear Segmented Tire, ω = 10.0 Hz, h_o = 0.5 ft, p_o = 45 psi.

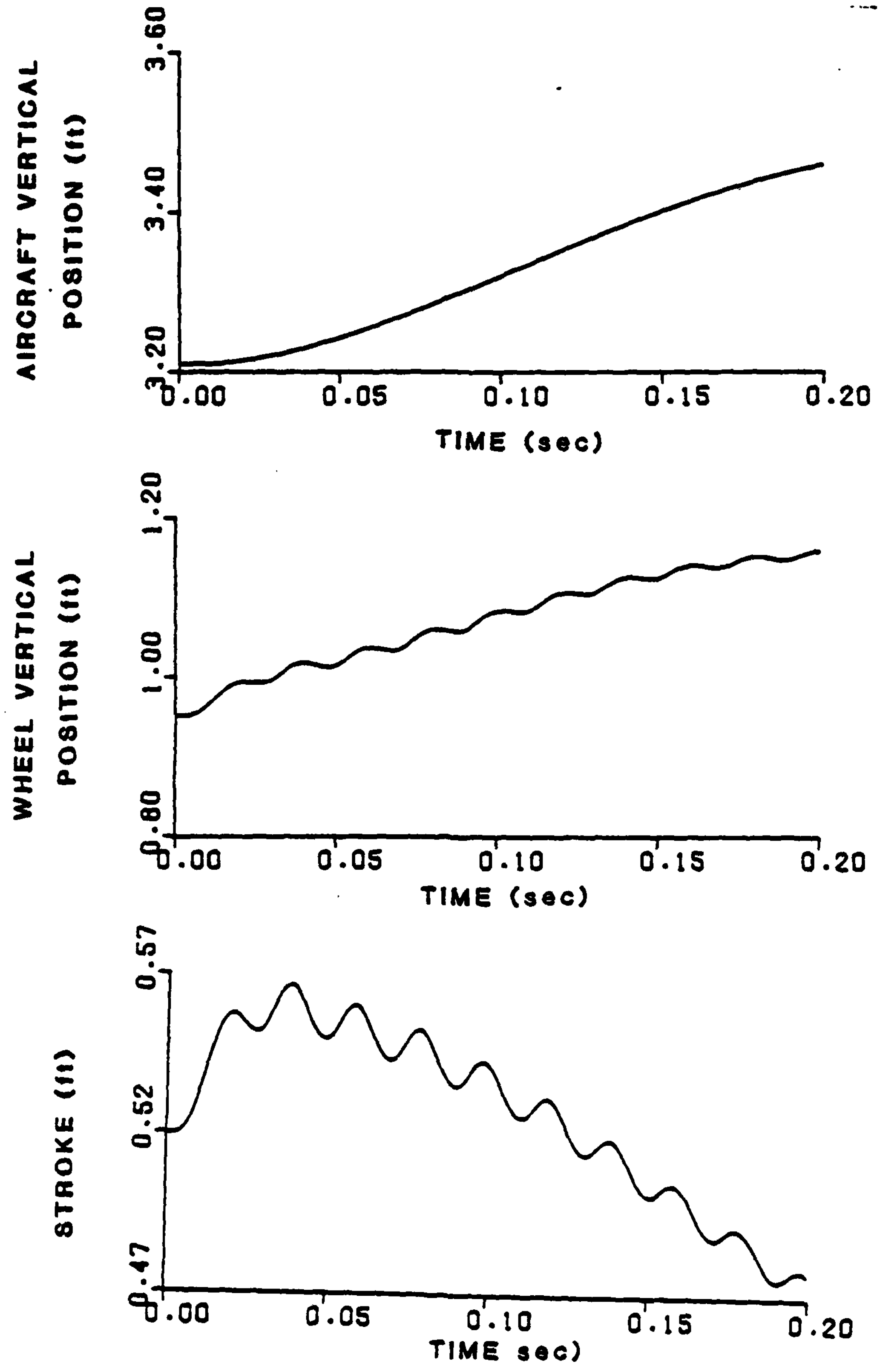


Figure 10a. System Response, 1-cos Profile, Linear Segmented Tire, ω = 50.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

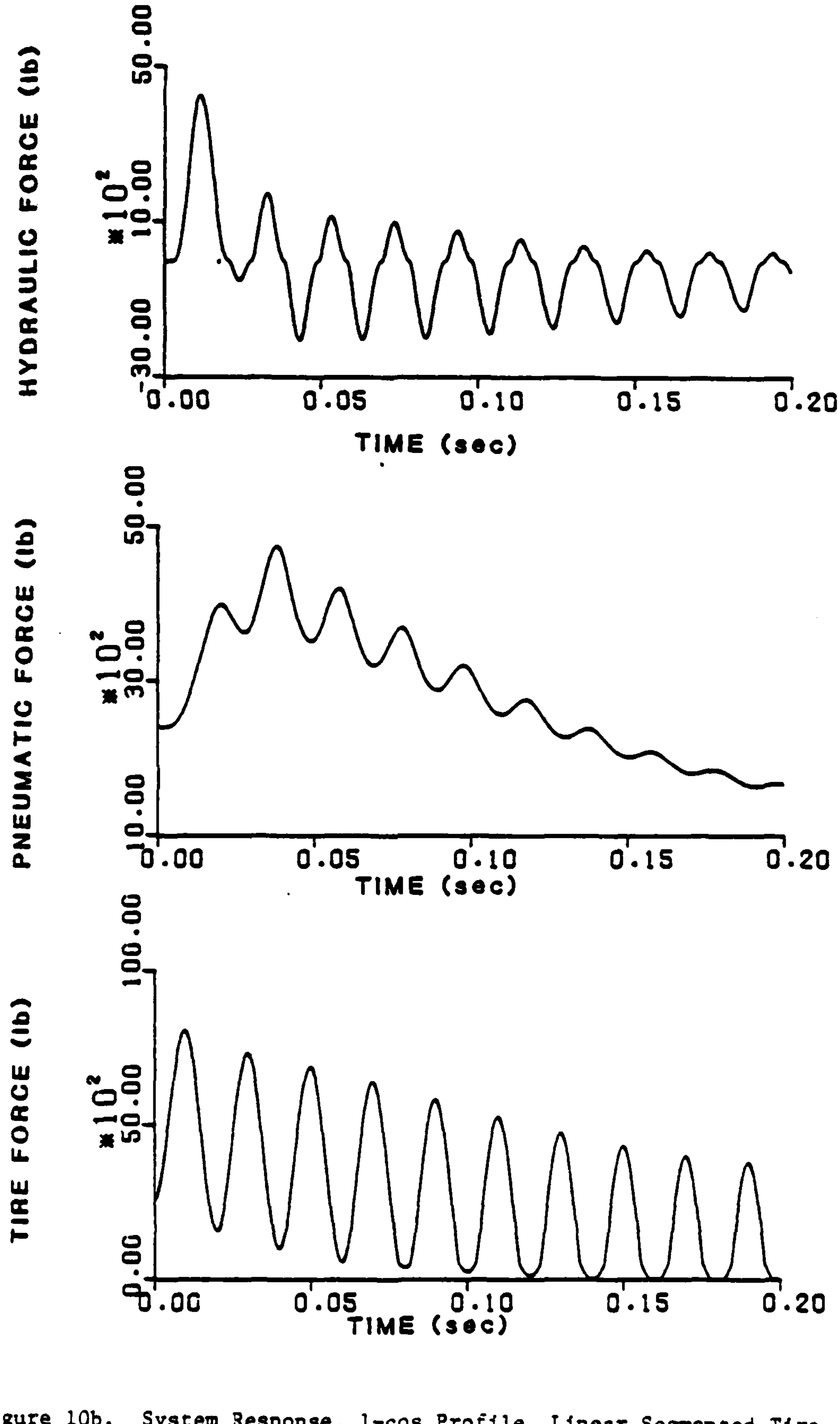


Figure 10b. System Response, 1-cos Profile, Linear Segmented Tire, ω = 50.0 Hz, h_o = 0.25 ft, p_o = 45 psi.

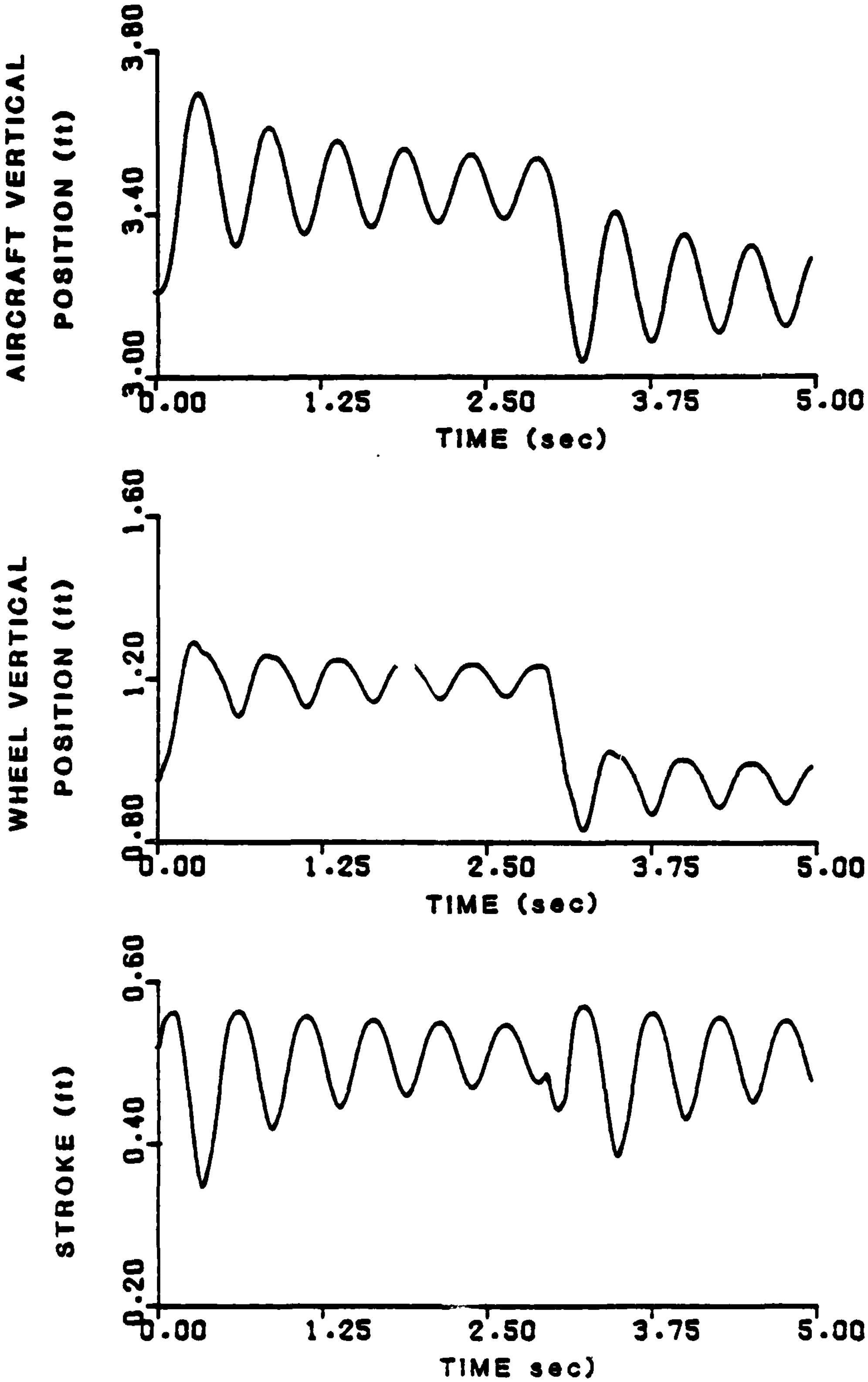


Figure 11a. System Response, Patch Profile, Linear Segmented Tire, $V_{a/c} = 25$ ft/sec, $h_o = 0.25$ ft, $p_o = 45$ psi.

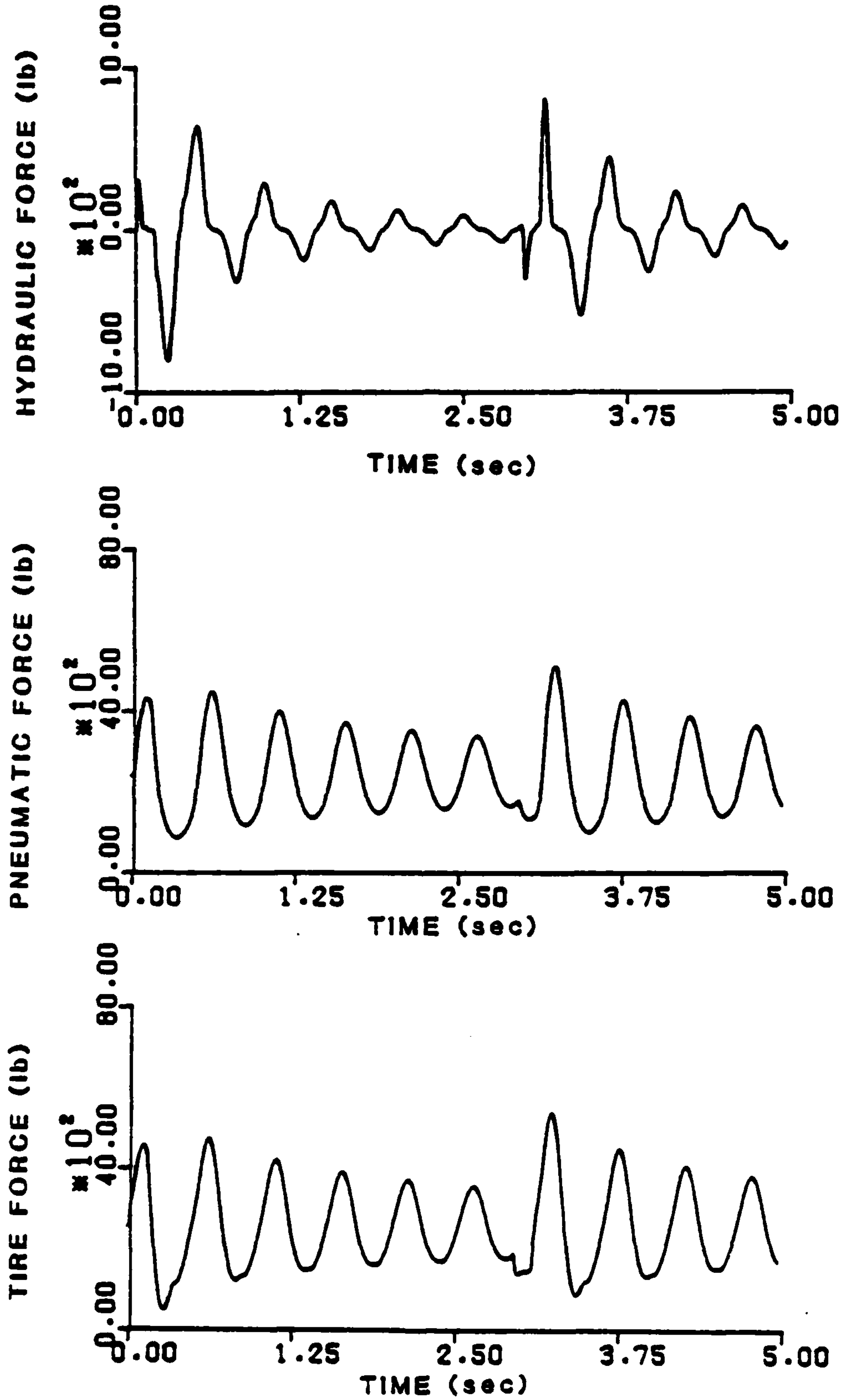


Figure 11b. System Response, Patch Profile, Linear Segmented Tire, $V_{a/c} = 25$ ft/sec, $h_o = 0.25$ ft, $p_o = 45$ psi.

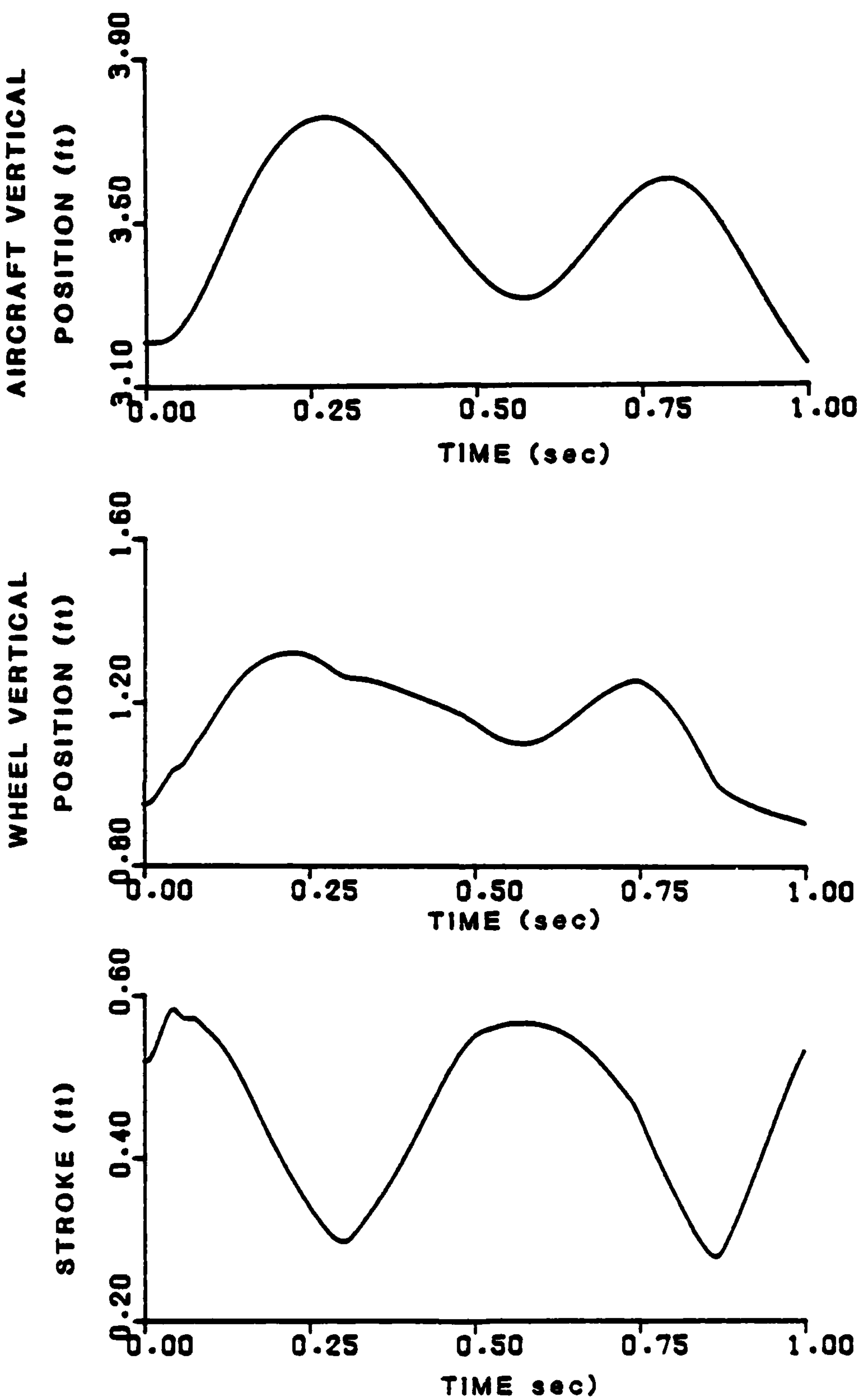


Figure 12a. System Response, Patch Profile, Linear Segmented Tire, $V_{a/c}$ = 100 ft/sec, h_o = 0.25 ft, p_o = 45 psi.

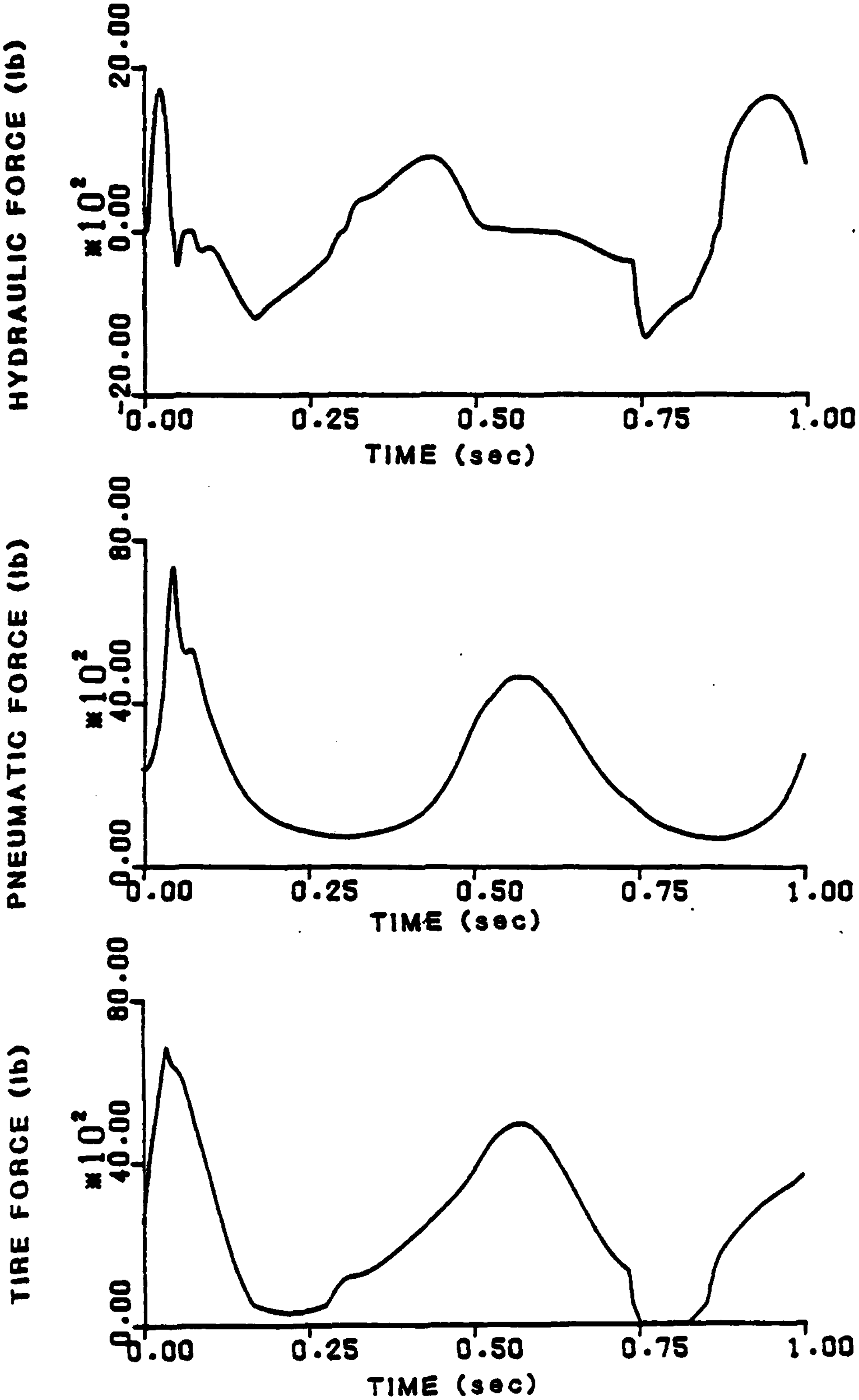


Figure 12b. System Response, Patch Profile, Linear Segmented Tire, $V_{a/c}$ = 100 ft/sec, h_o = 0.25 ft, p_o = 45 psi.

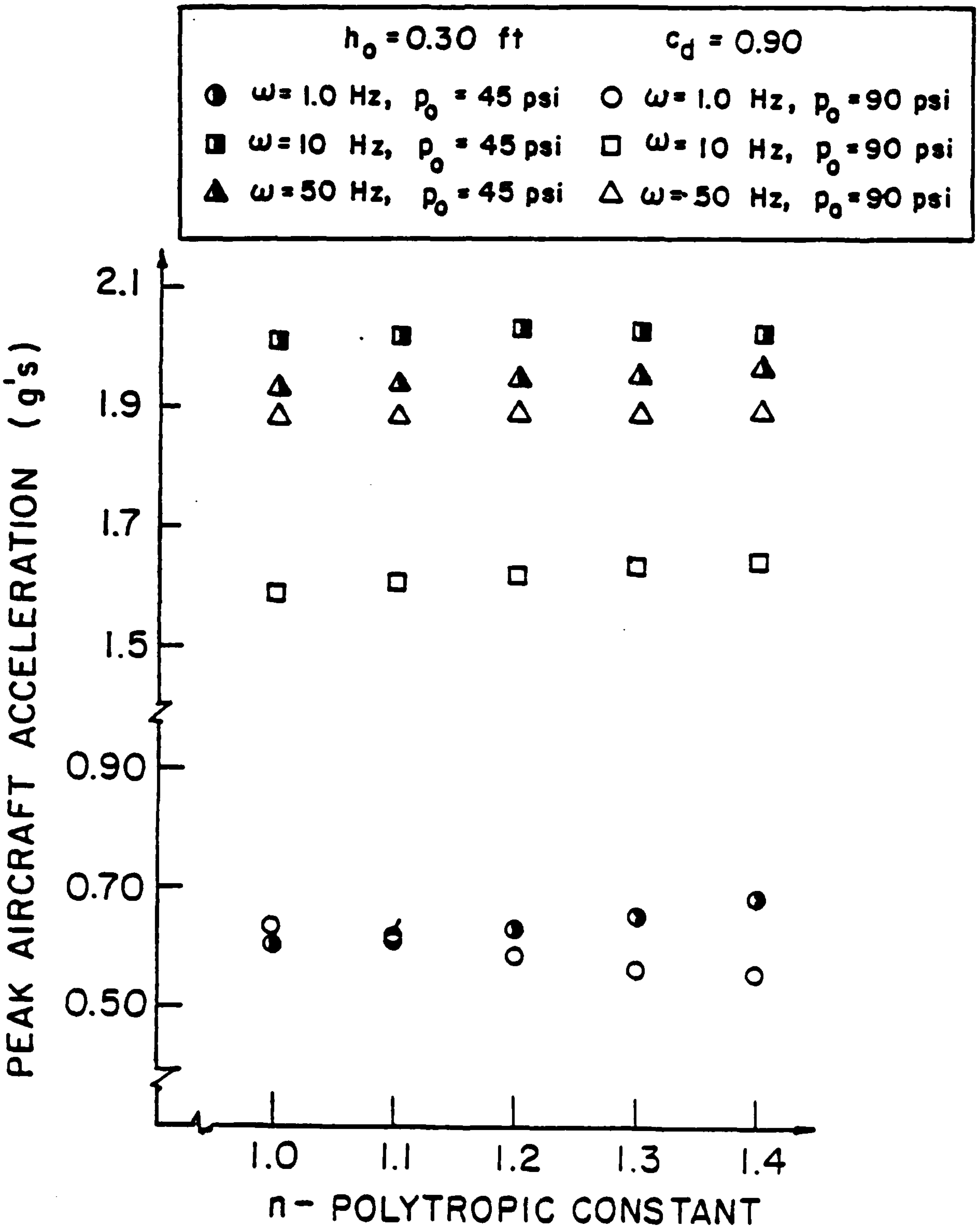


Figure 13a. Sensitivity of Aircraft Response to Polytropic Gas Constant

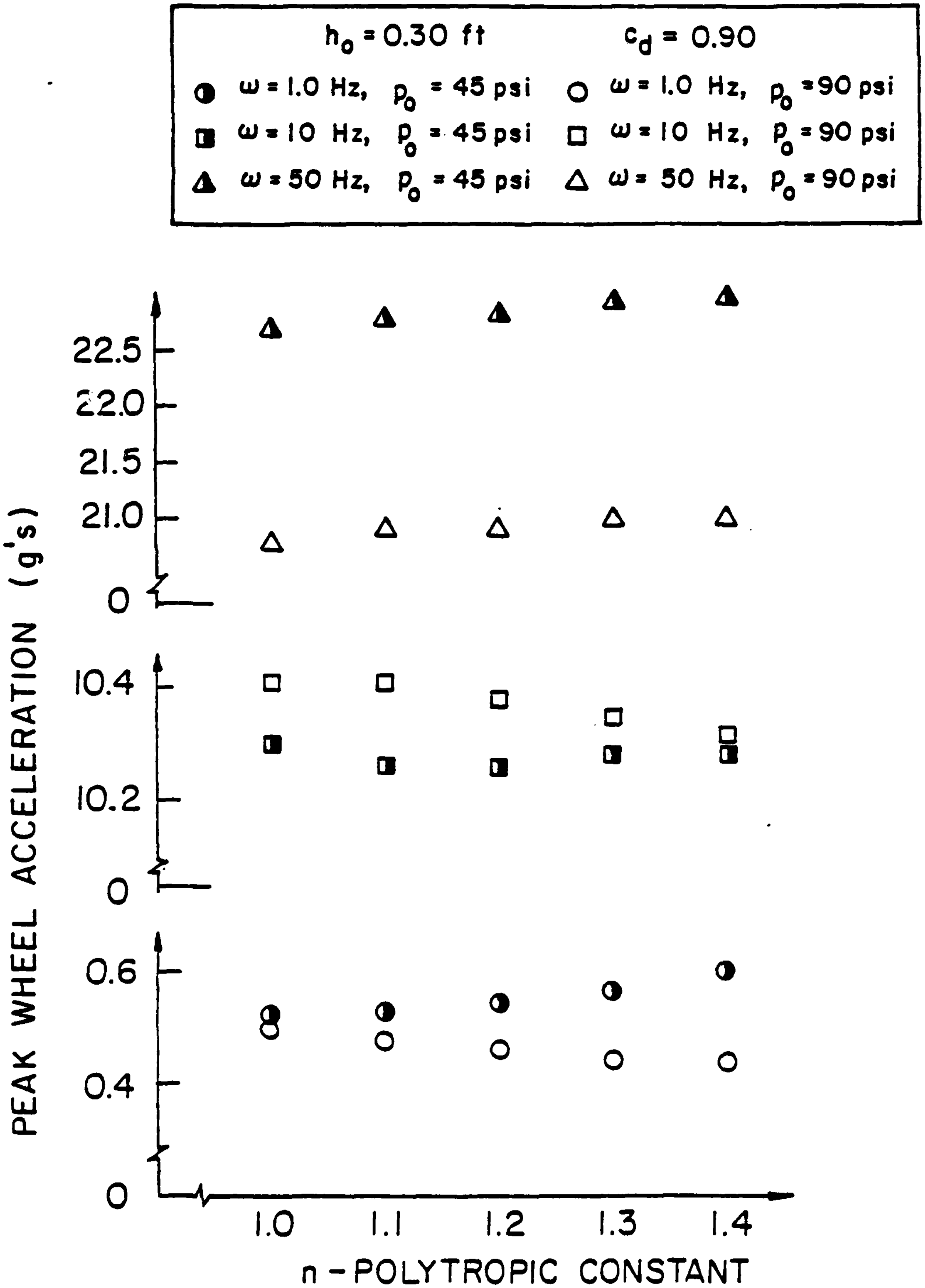


Figure 13b. Sensitivity of Wheel Response to Polytropic Gas Constant

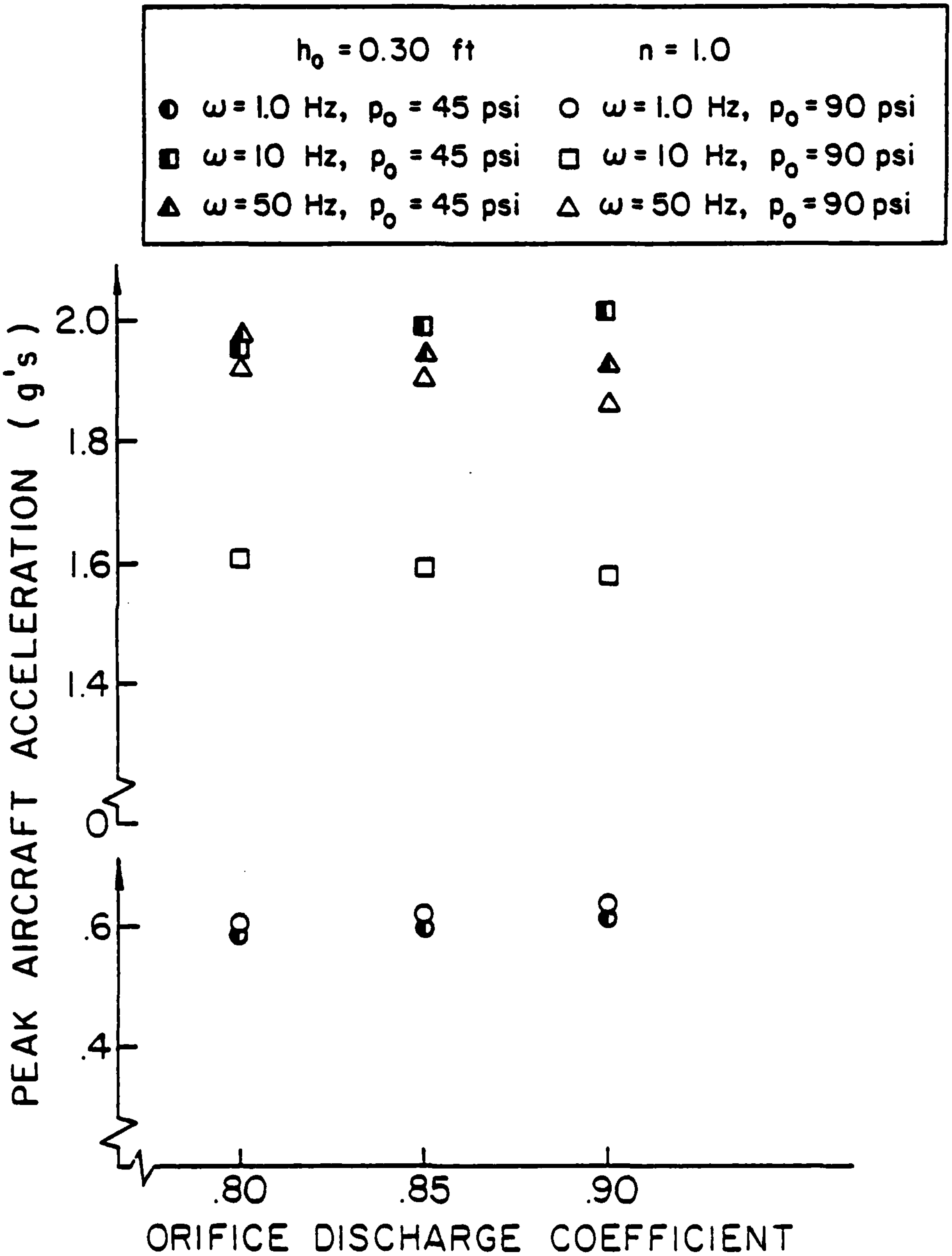


Figure 14a. Sensitivity of Aircraft Response to Orifice Discharge Coefficient

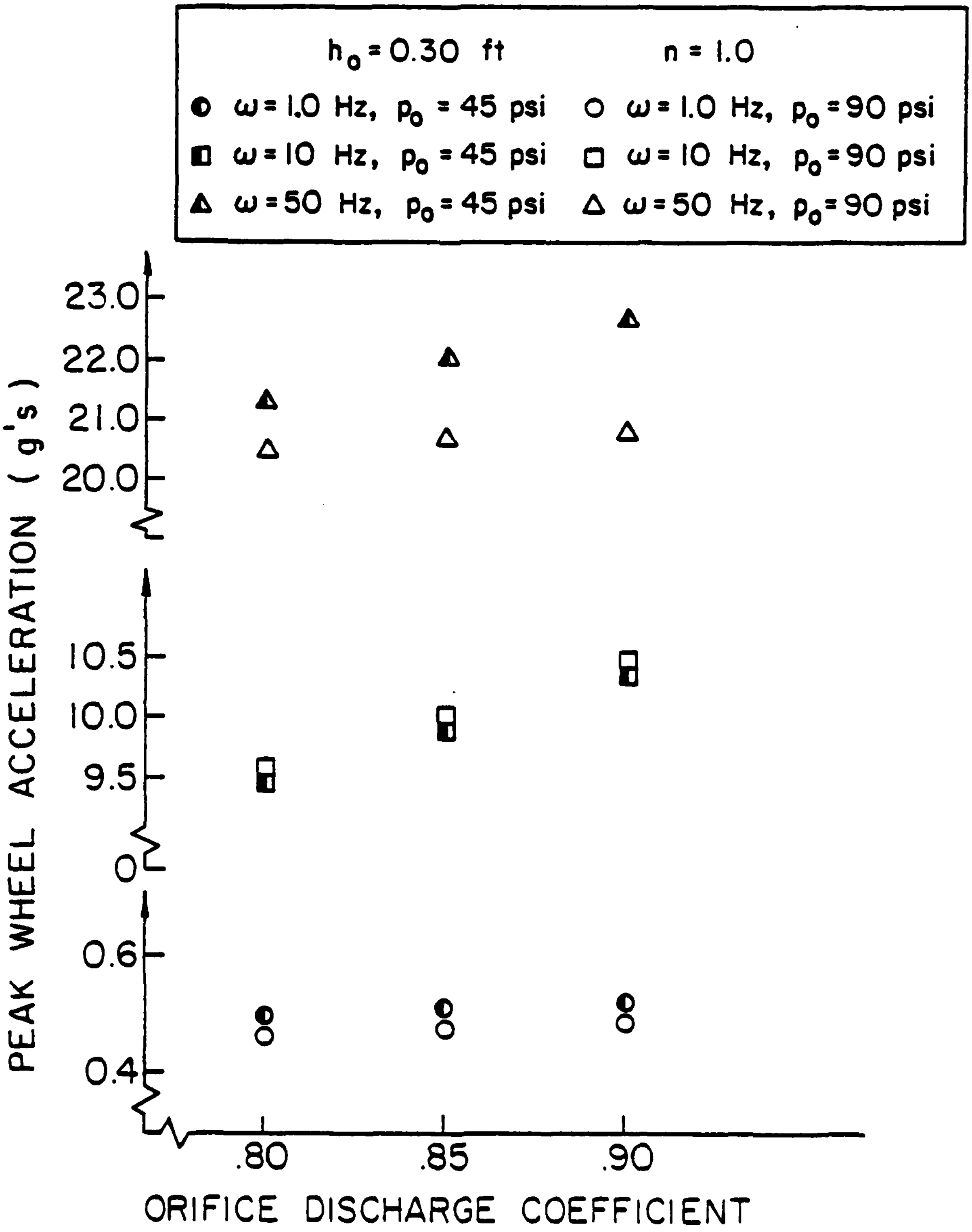


Figure 14b. Sensitivity of Wheel Response to Orifice Discharge Coefficient

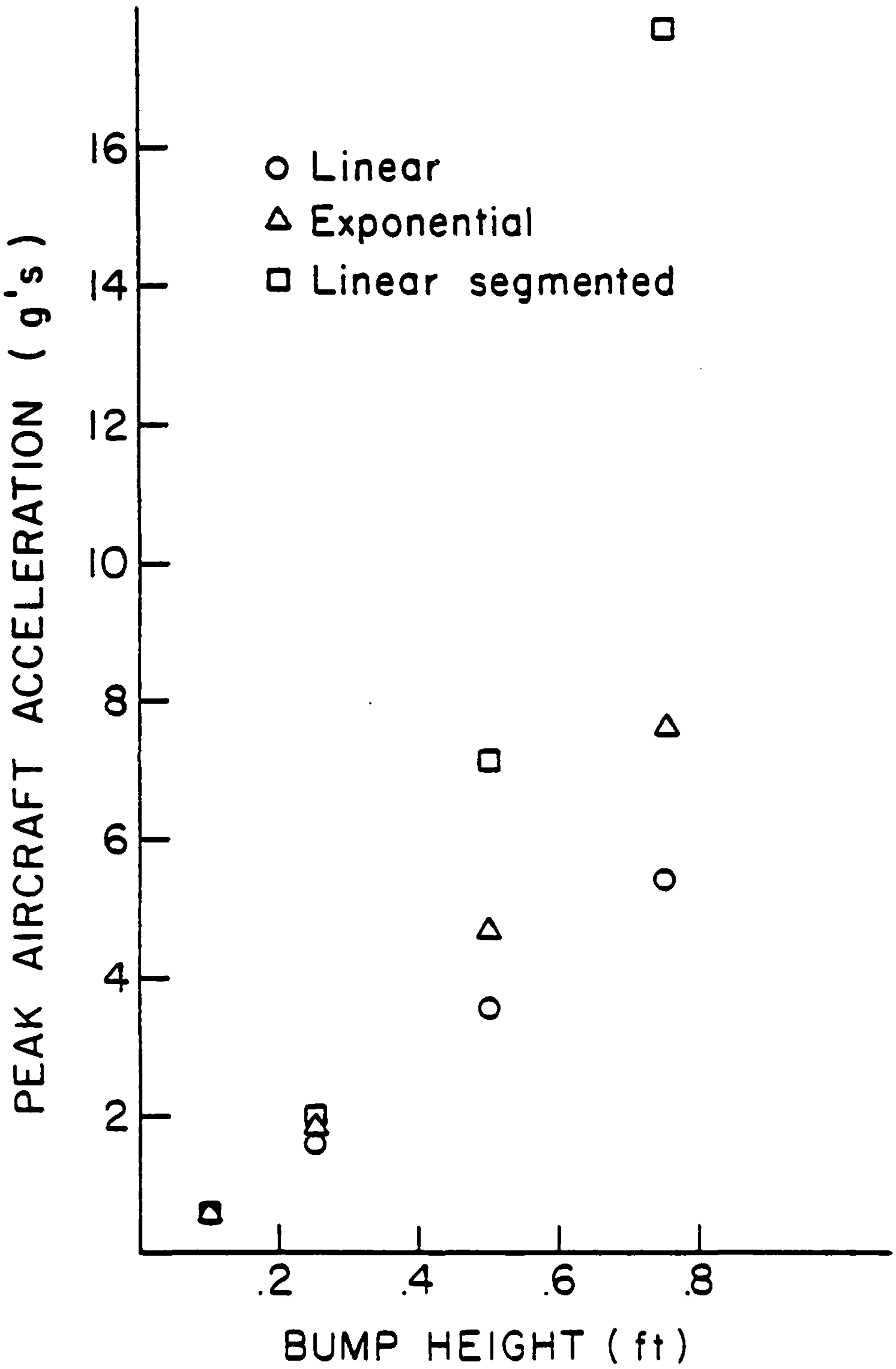


Figure 15a. Tire Model Influence 1-cos Profile, 10.0 Hz, $n = 1.0$, $C_d = 0.9$, $p_o = 45$ psi.

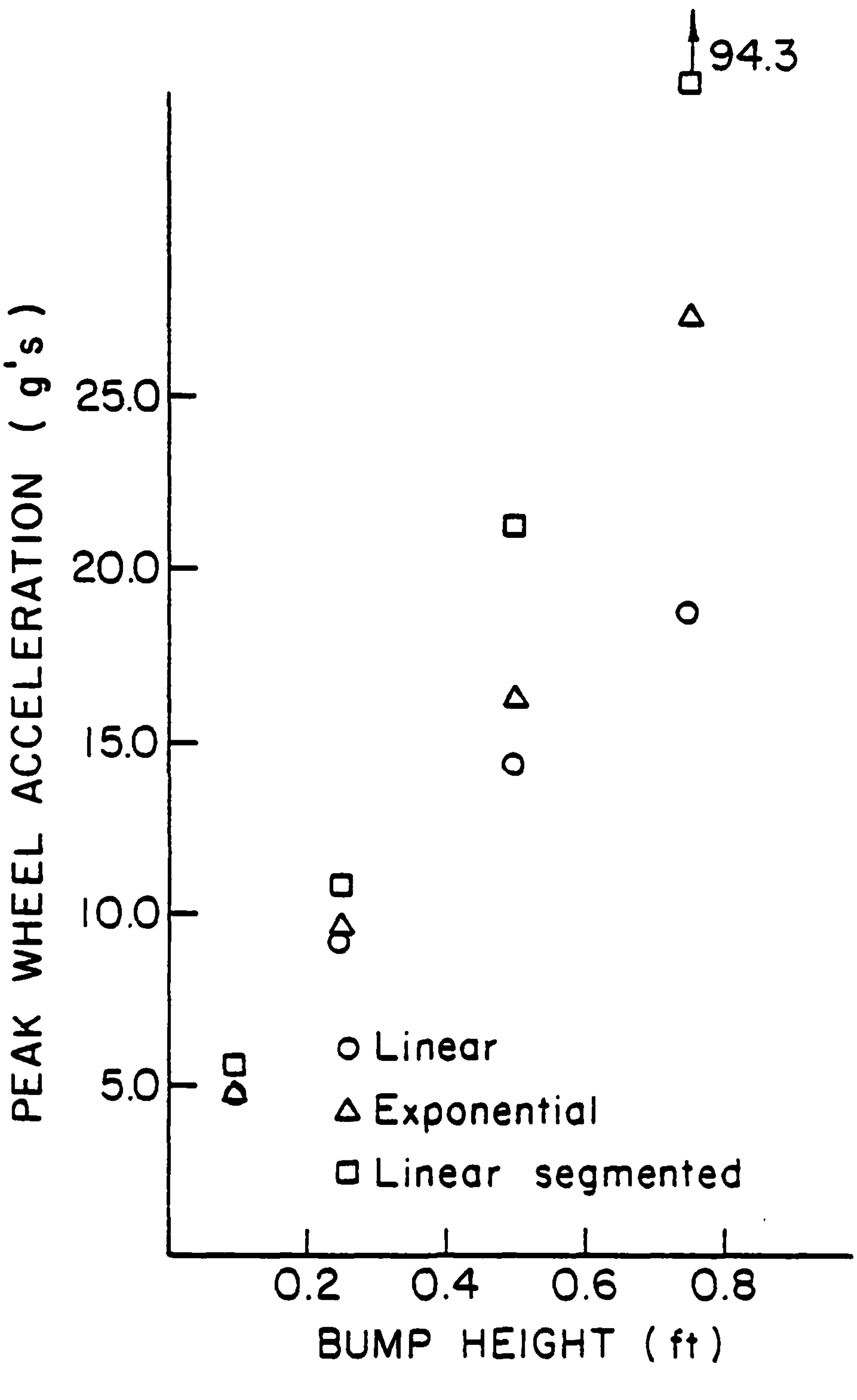


Figure 15b. Tire Model Influence, 1-cos Profile, 10.0 Hz, $n = 1.0$, $C_d = 0.9$, $p_o = 45$ psi.

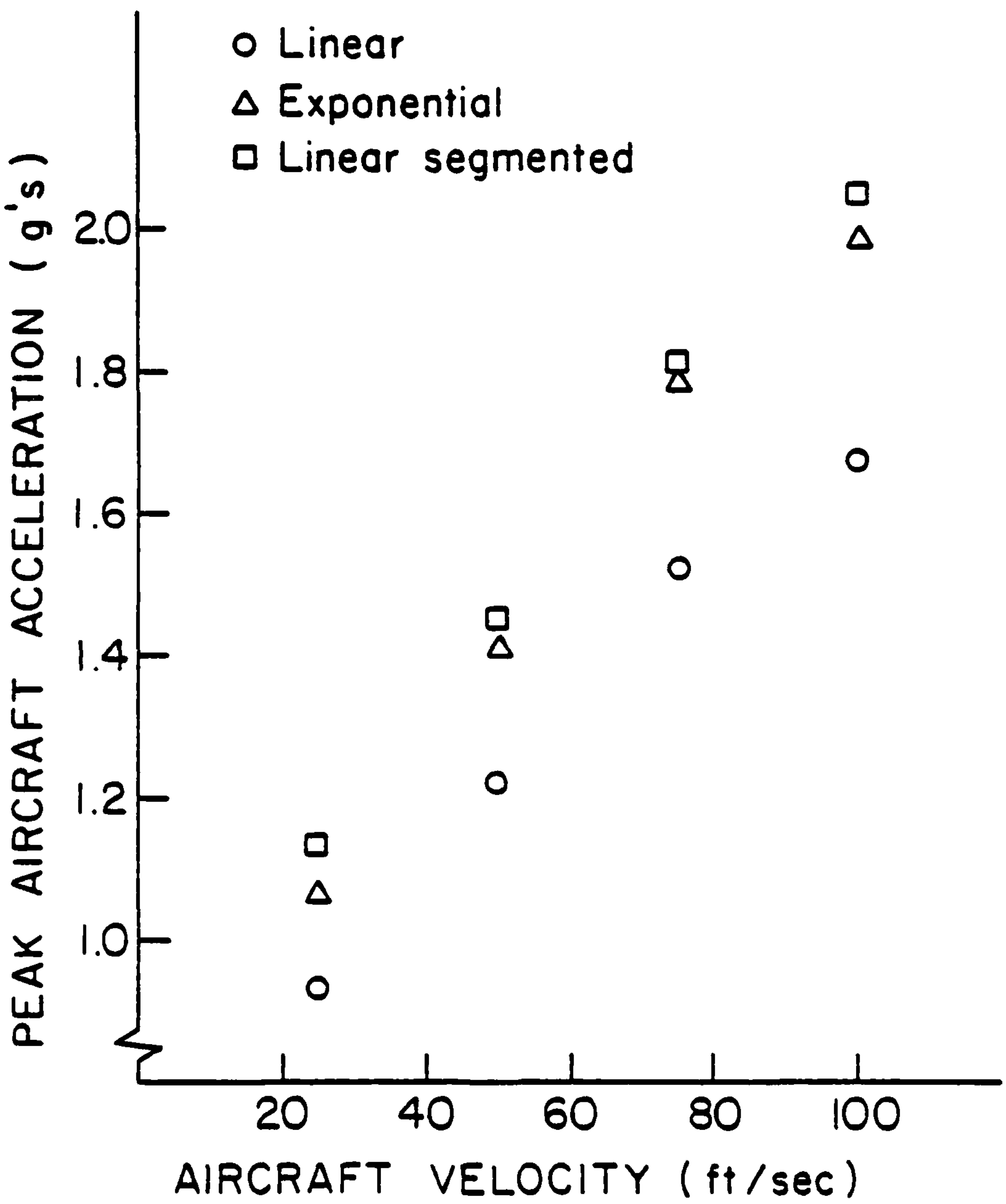


Figure 16a. Tire Model Influence, Patch Profile, h_o = 0.25 ft, C_d = 0.9, n = 1.0, p_o = 45 psi.

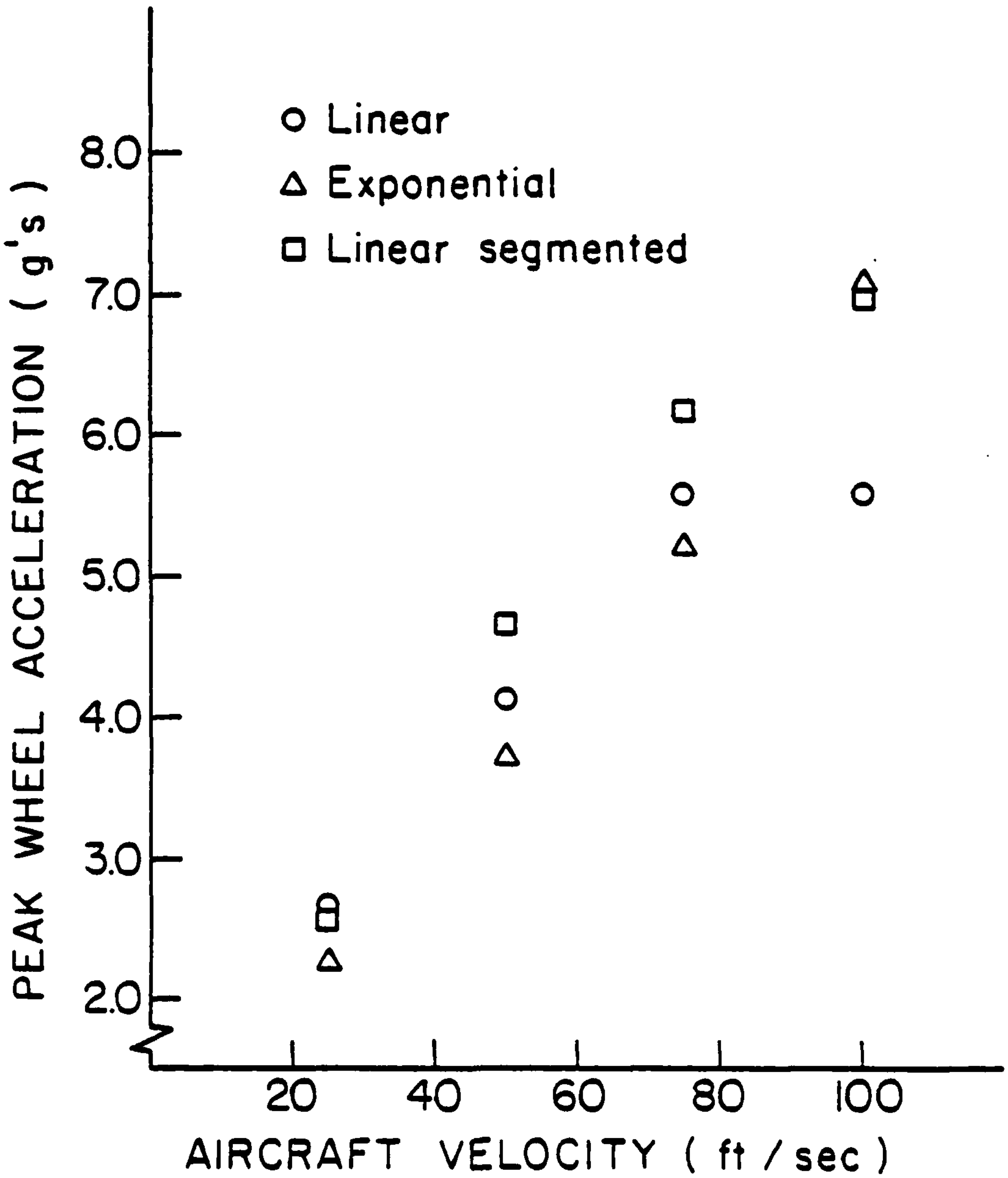


Figure 16b. Tire Model Influence, Patch Profile, h_o = 0.25 ft, C_d = 0.9, n = 1.0, p_o = 45 psi.

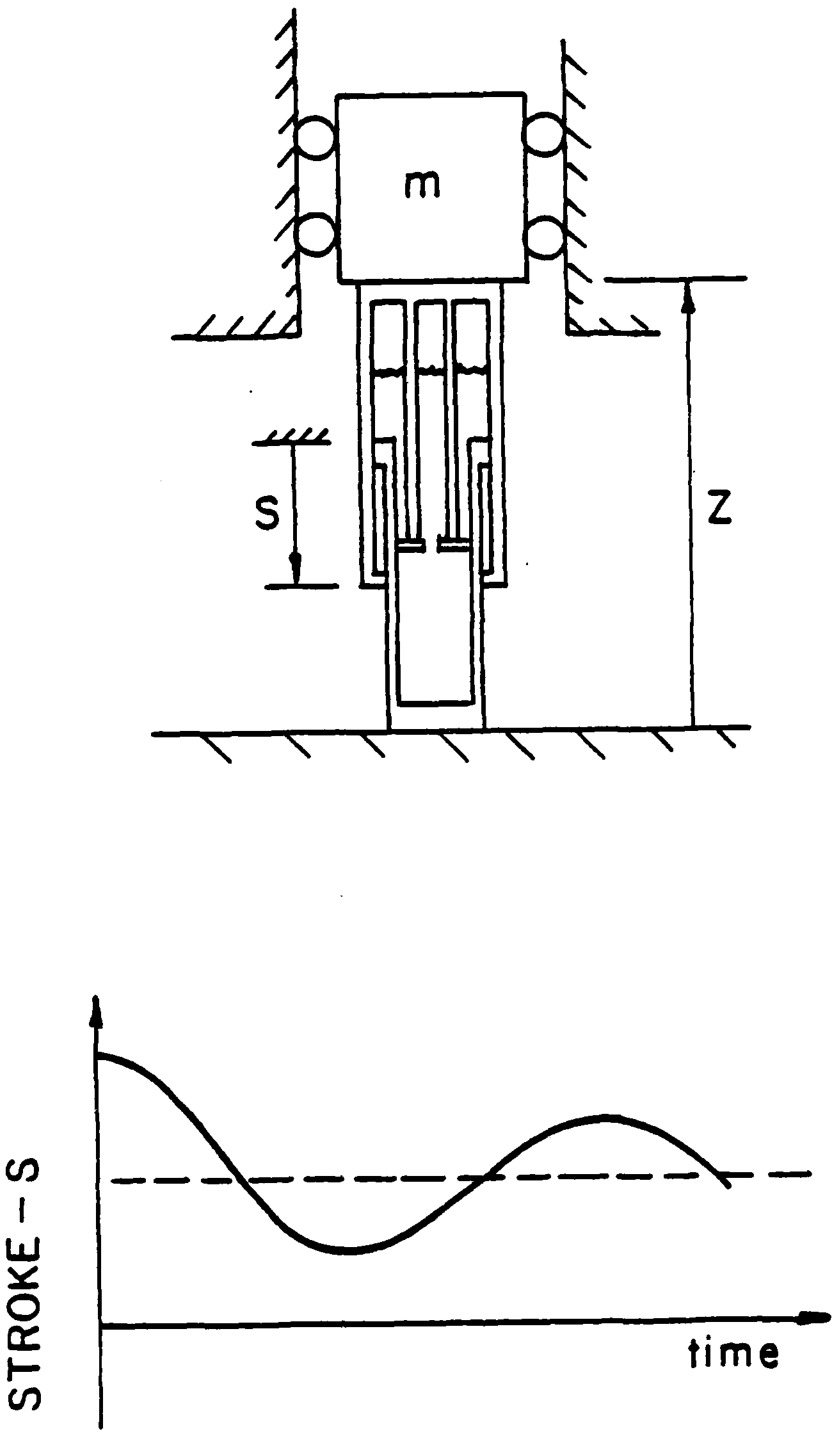


Figure 17. Single Degree of Freedom (Strut only) System and "Typical" Response

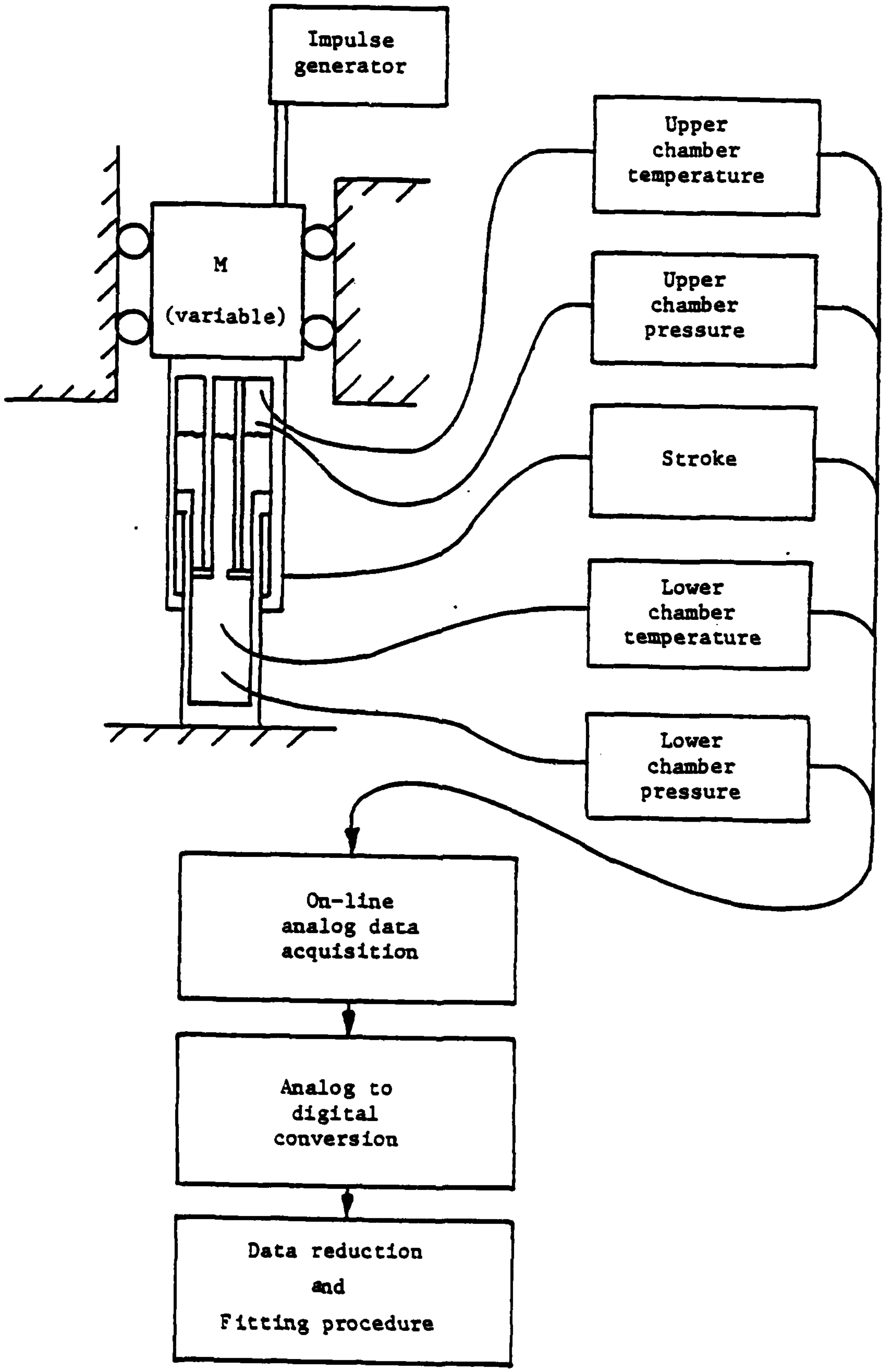


Figure 18. Schematic of Dynamic Test Facility and Instrumentation

APPENDIX A

AIRCRAFT TAXI DYNAMICS INDUSTRY SURVEY

The following questions were sent to twenty-nine major aircraft manufacturers and government laboratories. Responses were received from thirteen organizations; of that number, eleven provided comments on one or more of the survey questions. Those organizations which responded to the survey are also included following the survey questions.

Survey

1. Have you or your organization developed computer simulations to predict the dynamic response of an aircraft while it is operating in a taxi mode? Are any references available which describe the gear model in these simulations?

2. What kind of analytic model was used to predict the tire/wheel/gear contribution to the dynamic response?

   a) Have you performed an evaluation of the analytic model used for the tire/wheel/gear?

   b) How significant is the choice of the tire/wheel/gear model in the overall simulation?

   c) Do you feel the model is adequate for large amplitude dynamic response?

   d) Does the model have the capability of handling:

      1) Gear deformation?
      2) Frictional effects?
      3) Gas/fluid mixing?
      4) Drag loads in braking or non-braking operation?
      5) Others you may consider more important?

   e) What methods are used for determining the parameters (i.e., gas constant, discharge coefficient, etc.) used in the gear model?

   f) What are considered the most critical gear parameters? Why?

3. What do you think are the drawbacks or deficiences of the available gear model for predicting tire/wheel/gear contributions?

4. What criteria are used in evaluating the dynamic performance of the gear system? Can you predict failure of the gear system due to dynamic loadings?

5. Are any non-proprietary publications available to help in an assessment of the state of the art of landing gear modeling?

Responding Organizations

1. Northrop Corporation, Aircraft Division, Hawthorne, California
2. Boeing Company, Military Airplane Company, Seattle, Washington
3. BDM Corporation, Tyndall AFB, Florida
4. General Dynamics, Fort Worth Division, Fort Worth, Texas
5. McDonnell Douglas Corporation, McDonnell Aircraft Company, St. Louis, Missouri
6. Western Gear Corporation, Lynwood, California
7. Grumman Aerospace Corporation, Bethpage, New York
8. Beech Aircraft Corporation, Wichita, Kansas
9. Lockheed Corporation, Lockheed-Georgia Company, Marietta, Georgia
10. Rockwell International, North American Aircraft Division, Columbus, Ohio

APPENDIX B

OUTLINE OF NEWTONIAN ITERATION PROCEDURE

The concept of matching or "fitting" experimental data with analytic models is a classic form of analysis. Consider a dynamic system such as that represented by equation (B1) or other similar forms which can be written as:

$$F(\ddot{s}, \dot{s}, s, \underline{K}, t) = 0 \tag{B1}$$

The Newtonian iteration procedure provides a method for determining the set of constant parameters, $\underline{K}$, given the form of the function F. This is done by fitting numerically integrated solutions of the differential equations of motion to actual stroke versus time data. The vector $\underline{K}$ is composed of constants from the differential equation of motion: (from equation IV-1)

$$K_1 = \frac{\rho\,(A_{hy})^3}{2\,Cd^2\,A_{or}\,m} \quad , \qquad K_2 = n \tag{B2}$$

as well as initial conditions for the dynamic system:

$$K_3 = s(t=0) \quad , \qquad K_4 = \dot{s}(t=0) \tag{B3}$$

By defining the residual as the difference between the numerical solution to the differential equation and the experimental value of the stroke, the condition for the best fit is the parameter set $\underline{K}$ which provides a numerical solution which minimizes the sum of the square of the residuals. Given a set of experimental data, s_i, for each $t_i = 1, \ldots N$ where N is the total number of data points, the sum of the residuals' square can be expressed as:

$$SRS = \sum_{i=1}^{(N)} (s_i - \hat{s}_i)^2 \tag{B4}$$

The values for s_i are the analytic predictions for corresponding times, t_i.

To apply the least squares theory, the calculated position values are determined by expressing s_i as a Taylor series about some values calculated using initial estimates of the parameter vector, K_o:

$$s_i = s(t_i, \underline{K}_o) + \sum_{j=1}^{4} \left(\frac{\delta s}{\delta K_j}\right)_o \Delta K_j + \text{Higher Order Terms} \tag{B5}$$

where $\Delta K_j = K_j - K_{jo}$ (B6)

It is the correction, ΔK_j, to some first guess for the parameters which are used to improve the "fit" and provide the basis for an iteration procedure, resulting in satisfying the least squares criteria.

For equation (B4) to be a minimum, the following must be satisfied for each parameter:

$$\frac{\partial}{\partial K_k}(SRS) = 0 , \qquad k = 1, \ldots 4$$

Neglecting the higher order terms in the series expansion, equation (B5) is substituted into (B2) and the partial differentiation performed:

$$\sum_{i=1}^{N} 2\,[\hat{s}_i - s(t_i,\underline{K}_o) - \sum_{j=1}^{4} (\frac{\partial s}{\partial K_j})\,\Delta K_j]\,(\frac{\partial s}{\partial K_k}) = 0, \quad k = 1, \ldots 4 \qquad \text{(B7)}$$

In equation (B7), the unknowns are the partial derivatives $\partial s/\partial K_j$ and the parameter corrections ΔK_j. The derivatives can be found, again, by referring to the original differential equation of motion and defining the following:

$$P_k = \frac{\partial s}{\partial K_j} \qquad \text{(B8)}$$

Assuming that s is a continuous function of $\underline{K}$ and t, the higher order derivatives can be written as:

$$\frac{\partial \dot{s}}{\partial K_j} = \dot{P}_k , \qquad \frac{\partial \ddot{s}}{\partial K_j} = \ddot{P}_k .$$

Then, differentiation of the original equation of motion (B1) yields:

$$\frac{\partial F}{\partial \ddot{s}}\frac{\partial \ddot{s}}{\partial K_j} + \frac{\partial F}{\partial \dot{s}}\frac{\partial \dot{s}}{\partial K_j} + \frac{\partial F}{\partial s}\frac{\partial s}{\partial K_j} + \frac{\partial F}{\partial K_k} = 0$$

or

$$\frac{\partial F}{\partial \ddot{s}}\ddot{P}_k + \frac{\partial F}{\partial s}\dot{P}_k + \frac{\partial F}{\partial s}P_k = -\frac{\partial F}{\partial K_k} \qquad \text{(B9)}$$

At this point, the form of F must be set. For the simple case outlined in Section II of the report, the equation of motion of the system can be written as:

$$F = \ddot{s} + K_1 \dot{s} |\dot{s}| + \frac{p_o A_{pn}}{m} \left[\frac{V_o}{V_o - s A_{pn}} \right]^n - g = 0 \qquad \text{(B10)}$$

So, from equation (B10),

$$\frac{\partial F}{\partial \ddot{s}} = 1$$

$$\frac{\partial F}{\partial \dot{s}} = 2K_1 |\dot{s}|$$

$$\frac{\partial F}{\partial s} = \frac{n \acute{P}_o A_{pn}{}^2 (V_o)^n}{m(V_o - s A_{pn})^{n+1}}$$

Then, the variational equations are a set of second order ordinary differential equations:

$$\ddot{P}_k + 2K_1 |\dot{s}| P_k + \frac{n p_o A_{pn}{}^2 (V_o)^n}{m(V_o - s A_{pn})^{n+1}} P_k = - \frac{\partial F}{\partial K_k} \qquad \text{(B11)}$$

Numerical solutions of these equations will yield values of P_k at each time t_i. Initial conditions for the variation equations are directly determined from the definition of P_k:

$$P_3 (t = 0) = 1 , \qquad \dot{P}_4 (t = 0) = 1$$

with all others zero.

To use these numerical values of the partials required in the least squares conditions, the following is defined:

$$R_i = (\hat{s}_i - s_i)$$

Then equation (B7) can be written as:

$$\sum_{i=1}^{N} R_i P_k = \sum_{i=1}^{N} \sum_{j=1}^{N} P_j P_k \Delta K_j \qquad \text{for } k = 1 , \ldots 4$$

These four equations in the four unknown corrections ΔK_j can be written in matrix form:

$$\underline{b} = [A] \underline{x}$$

where $\underline{b} = [b_j]$ $\qquad b_j = \sum_{i=1}^{N} R_i P_j$

$\underline{x} = [x_j]$ $\qquad\qquad$ $x_j = \Delta K_j = K_j - K_{jo}$

$[A] = [a_{kj}]$ $\qquad\qquad$ $a_{kj} = \sum_{i=1}^{N} P_k P_j$

The system of normal equations from the least squares development has the solution:

$$\underline{x} = [A]^{-1} \underline{b}$$

which gives the new estimate of the parameter set:

$$K_j = K_{jo} + \Delta K_j$$

and is the basis for iterative procedure.

Given a set of initial approximations for the parameter set $\underline{K}$, the equation of motion (B10) and the variation equation (B11) can be numerically integrated. The least squares system of normal equations is formed and the parameter corrections determined. Corrected parameters are used as the new approximations, and the procedure repeated until the sum of the residuals squared is minimized. The minimum value of the residuals is determined by the truncation of data and experimental inaccuracies. Once the convergence criteria are satisfied, the parameters C_d and n are known for a given set of experimental data.

REFERENCES

1. A. G. Gerardi, "Digital Simulation of Flexible Aircraft Response to Symmetrical and Asymmetrical Runway Roughness," Air Force Wright Aeronautical Laboratories, AFFDL-TR-77-37, August 1977.

2. C. Mitchell, "Some Measured and Calculated Effects of Runway Unevenness on a Supersonic Transport Aircraft," The Aeronautical Journal of the Royal Aeronautical Society, Vol. 75, p. 339, May 1971.

3. M. K. Wahi, "Oleopneumatic Shock Strut Dynamic Analysis and Its Real Time Simulation," AIAA Journal of Aircraft, Vol. 13, No. 4, p. 303, April 1976.

4. F. C. Allen, and L. B. Mosby,"An Investigation of Aircraft Loads Developed During Landings on Rough Terrain," Canadian Aeronautics and Space Journal, Vol. 11, No. 2, p. 43, February 1965.

5. D. M. Rehnder and L. B. Mosby,"Ground Loads for Aircraft with Multiple Wheels and Multiple Landing Gear with Requirements for Semi-improved Field Operations" Canadian Aeronautics and Space Journal, Vol. 12, No. 11, p. 359,November 1966.

6. R. R. Deakins, G. A. Pierce and R. W. Gorgeware, "An Analytical Technique for the Determination of Landing Gear Loads on the A3J-1 Airplane During Arrested Landing," North American Aircraft Division, Columbus Plant, Rockwell International Corp. Report No. NA61H-639, February 1962.

7. C. K. Butterworth and D. E. Boozer,Jr.,"C-141A Computer Code for Runway Roughness Studies," AFWL-TR-70-71, Air Force Weapons Laboratory, Kirtland, AFB, New Mexico, August, 1970.

8. C. C. Tung, J. Penzien and R. Horonjeff, "The Effect of Runway Unevenness on the Dynamic Response of Supersonic Transports," NASA CR-119, October 1964.

9. E. Schnitzer, "Band Pass Shock and Vibration Absorbers for Application to Aircraft Landing Gear," NACA TN-3803, October 1956.

10. C. L. Kirk, "The Random Heave-pitch Response of Aircraft to Runway Roughness," The Aeronautical Journal of the Royal Aeronautical Society, Vol. 75, p. 476, July 1971.

11. C. L. Kirk and P. J. Perry,"Analysis of Taxiing Induced Vibrations by the Power Spectral Density Method," The Aeronautical Journal of the Royal Aeronautical Society, Vol. 75, p. 182, March 1971.

12. C. Venkatesan and V. Krishnan, "Stochastic Modeling of an Aircraft Traversing a Runway Using Time Series Analysis," AIAA Journal of Aircraft, Vol. 18, No. 2, February 1981.

13. O. Kropac and J. Sprinc, "Interaction between Aircraft and Runway from the Point of View of Vehicle Dynamics," Polish Academy of Science, Nonlinear Vibration Problems, Vol. 14, p. 393, 1973.

14. J. C. Houbolt, J. H. Walls and R. F. Smiley, "On Spectral Analysis of Runway Roughness and Loads Developed During Taxiing" NACA TN-3484, July 1955.

15. N. S. Silsby, "An Analytical Study of Effects of Some Airplane and Landing-Gear Factors on the Response to Runway Roughness with Application to Supersonic Transports," NASA TN-1492, December 1962.

16. B. D. VanDeusen, "A Statistical Technique for the Dynamic Analysis of Vehicles Traversing Rough Yielding and Non-Yielding Surfaces," NASA CR-659, March 1967.

17. R. A. DeYoe, "Active Control for Aircraft Landing Gear," M.S. Thesis, Air Force Institute of Technology, Wright Patterson AFB, Ohio, June 1970.

18. W. Flugge, "Landing-Gear Impact," NACA TN-2743, October 1952.

19. A. J. Yorgiadis, "Graphical Analysis of Performance of Hydraulic Shock Absorbers in Aircraft Landing Gears," Journal of the Aeronautical Sciences, Vol. 12, No. 4, p. 421, October 1945.

20. B. Milwitzky and D. C. Lindquist,"Evaluation of the Reduced Mass Method of Representing Wing Lift Effects in Free Fall Drop Tests of Landing Gears," NACA TN-2400, July 1951.

21. J. H. Walls, "Investigation of the Air-Compression Process During Drop Tests of an Oleo-Pneumatic Landing Gear," NACA TN-2477, September 1951.

22. J. H. Walls "An Experimental Study of Orifice Coefficients, Internal Strut Pressures and Loads on a Small Oleo-Pneumatic Shock Strut," NACA-TN-3426, April 1955.

23. B. Milwitzky, and F. E. Cook,"Analysis of Landing Gear Behavior", NACA-TR-1154, 1953. (Supersedes NACA TN 2755)

24. W. C. Hurly, "A Study of the Response of an Airplane Landing Gear Using the Differential Analyzer," Journal of the Aeronautical Sciences, Vol. 17, No. 12, December 1950.

25. M. K. Wahi, "Oil Compressibility and Polytropic Air Compression Analysis for Oleopneumatic Shock Struts," AIAA Journal of Aircraft, Vol. 13, No. 7, July 1976.

26. C. Venkateson, "Comparison of Linear and Nonlinear Dampers for Landing Gear at Touch Down," AIAA Journal of Aircraft, Vol. 12, No. 10, October 1975.

27. C. Venkatesan and R. Krishnan, "Dual-Phase Damping in a Landing Gear at Touch Down," AIAA Journal of Aircraft, Vol. 12, No. 10, October 1975.

28. L. C. Cheng, M. E. Clark and J. M. Robertson, "Numerical Calculations of Oscillating Flow in the Vicinity of Square Wall Obstacles in Plane Conduits", Journal of Biomechanics, Vol. 5, p. 467, 1972.

29. R. S. Eikenberry, "Analysis of Angular Motion of Missiles," SC-CR-70-6051, Sandia Laboratories Albuquerque, New Mexico, February 1970.

30. T. R. Goodman, "System Identification and Prediction - An Algorithom Using a Newtonian Iteration Procedure," Quart. of Applied Math., Vol. 24, No. 3, October 1966.

31. G. T. Chapman, D. B. Kirk, "A New Method for Extracting Aerodynamic Data From Free Flight Data," AIAA Paper 69-134, January 1969.

32. S. M. Batill, C. W. Ingram, "Determining Aircraft Stability Coefficients from Dynamic Motions," AIAA Journal of Aircraft, Vol. 8, No. 11, November 1971.

33. S. M. Batill, C. W. Ingram, "An Investigation of Unsteady Aerodynamics on an Oscillating Airfoil," AIAA Paper 73-318, March 1973.

$n = 1.0$, $C_d = 0.9$, $p_o = 45$ psi.